MECH 230 Dynamics
Homework 4

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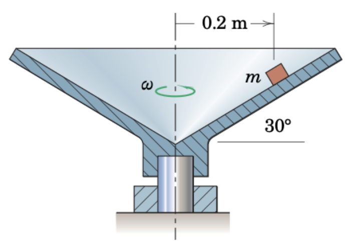
Due Wednesday October 2, 2024

1. Read the problem statement of MKB 3/67.

3/67 **The small object is placed on the inner surface of the conical dish at the radius shown. If the coefficient of static friction between the object and the conical surface is 0.30, for what range of angular velocities ω about the vertical axis will the block remain on the dish without slipping? Assume that speed changes are made slowly so that any angular acceleration may be neglected.**



PROBLEM 3/67

2. Set up a cylindrical-polar coordinate system whose origin is taken to be at the tip of the cone.
3. The position vector to the object of mass m can be written as

$$\mathbf{r} = r\mathbf{e}_r + z\mathbf{E}_z = r\mathbf{e}_r + r\tan(30^\circ)\mathbf{E}_z.$$

   Write the velocity and acceleration (vectors) of the particle of mass m.
4. How do the position, velocity, and accelerations expressions from part 3 simplify when the mass m is sticking on the cone?
5. For notational convenience, we also define the corotational unit vectors.

$$\begin{aligned}\mathbf{e}_1 &= \cos(30^\circ)\mathbf{e}_r + \sin(30^\circ)\mathbf{E}_z\\ \mathbf{e}_2 &= -\sin(30^\circ)\mathbf{e}_r + \cos(30^\circ)\mathbf{E}_z.\end{aligned}$$

Add these vectors to your sketch of the cone.

Corotational vectors are just vectors that are fixed to a moving vector. For example, $\{\mathbf{e}_1, \mathbf{e}_2\}$ are fixed to the moving cone and co-rotate with it.

6. Write $\mathbf{r}$ in the $\{\mathbf{e}_1, \mathbf{e}_2\}$ basis.
7. Draw the free body diagram of the particle m and write the expressions of the forces acting on the particle where applicable.

   In determining the normal and friction force directions, remember that the particle is resting on a conical *surface*. Take the friction force to be static in this part.
8. Write the balance of linear momentum equation (BoLM, also known as $\mathbf{F} = m\mathbf{a}$) in vector form. Take the friction force to be static in this part.
9. Take appropriate projections of the BoLM to find expressions for the normal and static friction forces.
10. Use the static friction criterion to find an expression for the range of angular velocities ω for which the block will remain on the cone without slipping.
11. If the mass m were to slip on the cone, what would be the expression of the kinetic friction force acting on it?
12. In this case, how would you obtain the equations of motion of the mass m.