* HW4 due today.
* HW5 due in a week (covers both W–E theorem & Angular Momentum)

Previously on Dynamics:

- Power $= \underline{P} \cdot \underline{v}$
- Work $W_{\underline{P},AB} = \int_{t_A}^{t_B} \underline{P} \cdot \underline{v}\, dt = \int_{\underline{r}_A}^{\underline{r}_B} \underline{P} \cdot d\underline{r}$
- Kinetic energy $T = \frac{1}{2} m \underline{v} \cdot \underline{v}$
- Work–Energy theorem
$$\begin{cases} \dot{T} = \underline{F} \cdot \underline{v} \\ T_B - T_A = W_{\underline{F},AB} \Longleftrightarrow E_B - E_A = W_{\underline{F}_{nc},AB} \end{cases}$$
- some forces are conservative $\underline{F}_c$. Their work is $W_{\underline{F}_c,AB} = U_A - U_B$
  - constant force $\underline{C}$ (eg. weight)
    $U_{\underline{C}} = -\underline{C} \cdot \underline{r}$
  - spring force $\underline{F}_s = -k\varepsilon \dfrac{\underline{r} - \underline{r}_A}{\|\underline{r} - \underline{r}_A\|}$
    $U_s = \frac{1}{2} k \varepsilon^2$
  - gravitational force (see set 8)

Today on Dynamics: Linear Momentum, Angular Momentum & their conservations.

linear momentum $\underline{G} = m\underline{v}$

BoLM $\boxed{\underline{F} = m\underline{a}} = \dot{\underline{G}}$ (if the mass is constant)

$$\boxed{\int_{t_A}^{t_B} \underline{F}\, dt = \underline{G}(t = t_B) - \underline{G}(t = t_A)}$$

$$\int_{t_A}^{t_B} \underline{F}\, dt = \underline{G}_B - \underline{G}_A = m\underline{v}_B - m\underline{v}_A \Longrightarrow \text{more general.}$$

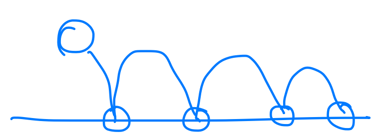
$\underline{a} = \dfrac{d\underline{v}}{dt}$

$\underline{a}$ is not defined at instances when $\underline{v}$ is not differentiable.

Linear impulse of a force.

Linear impulse – linear momentum equation

Time integral of the BoLM.



Conservation of Linear Momentum

quantity is not changing in time.

$\underline{G}$ is conserved $\Rightarrow \dot{\underline{G}} = \underline{0} \Rightarrow \underline{F} = \underline{0}$

$$\underline{G}_A = \underline{G}_B$$

sometimes $\underline{G}$ is not conserved (completely), but $\underline{G}$ might be conserved in a certain direction $\underline{c}$.

$\boxed{\underline{G}\cdot\underline{c} \text{ is conserved}}$

$$\Rightarrow \quad \overline{\underline{G}\cdot\underline{c}}^{\,\bullet} = \frac{d}{dt}(\underline{G}\cdot\underline{c}) = 0$$

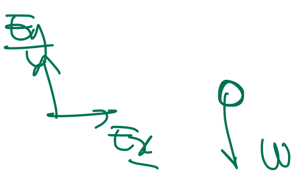
$$\dot{\underline{G}}\cdot\underline{c} + \underline{G}\cdot\dot{\underline{c}} = 0$$

$$\text{iff} \longrightarrow \boxed{\underline{F}\cdot\underline{c} + \underline{G}\cdot\dot{\underline{c}} = 0} \Longleftarrow$$

if $\underline{c}$ is a constant vector (eg. $\underline{E}_x, \underline{E}_y, \underline{E}_z, \ldots$) $\cancel{\underline{e}_r}, \cancel{\underline{e}_\theta}$

This result simplifies $\boxed{\underline{G}\cdot\underline{c} \text{ is conserved} \Leftrightarrow \underline{F}\cdot\underline{c} = 0} \Longleftarrow$

Eg. Projectile motion. Is $\underline{G}$ conserved ?

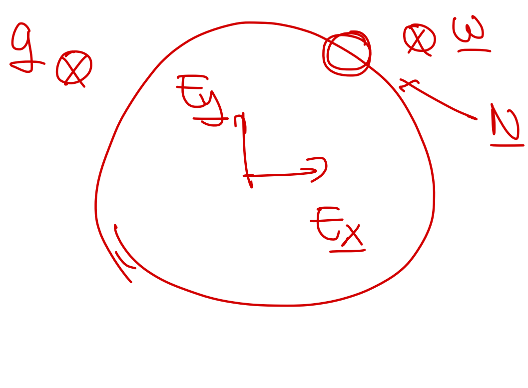


$$\underline{F} = \boxed{-mg\underline{E}_y = \dot{\underline{G}}}$$

$$\dot{\underline{G}}\cdot\underline{E}_x = 0 \Rightarrow \underline{G}\cdot\underline{E}_x = \text{const.}$$

Linear momentum in the x-direction is conserved.

eg.



$$\underline{w} = -mg\underline{E}_z$$

$$\underline{N} = N_r\underline{e}_r + N_z\underline{E}_z$$

$$\underline{G}\cdot\underline{e}_\theta = \text{const} \; ?$$

no! $\underline{e}_\theta$ is not constant if $\underline{F}\cdot\underline{e_\theta} + \underline{G}\cdot(-\dot{\theta}\,\underline{e_r}) = 0$

$\Rightarrow$ then $\underline{G}\cdot\underline{e_\theta} = \text{const.}$

Here, $\underbrace{\cancel{\underline{F}\cdot\underline{e_\theta}}}_{0} + m(\underbrace{\cancel{\dot{r}\,\underline{e_r}}}_{0} + r\dot{\theta}\,\underline{e_\theta})\cdot(-\dot{\theta}\,\underline{e_r})$

$= 0$

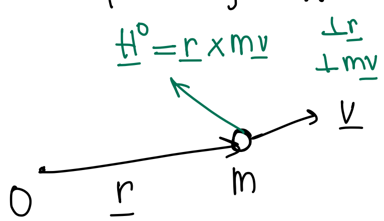
so $\underline{G}\cdot\underline{e_\theta}$ is conserved.

Angular Momentum

$\underline{H}^O = \underline{r}\times m\underline{v}$

⚠ Always specify about which point you are calculating your angular momentum.

The angular momentum of a particle about pt O is the position vector from O to the particle, crossed with $m\underline{v}$.



$$\dot{\underline{H}}^O = \frac{d}{dt}(\underline{r}\times m\underline{v})$$
$$= \underbrace{\cancel{\underline{v}\times m\underline{v}}}_{0} + \underline{r}\times m\underline{a}$$
$$= \underline{r}\times m\underline{a}$$

$$\frac{d}{dt}(\underline{a}\times\underline{b}) = \dot{\underline{a}}\times\underline{b} + \underline{a}\times\dot{\underline{b}}$$

BoLM $\quad \underline{F} = m\underline{a}$

$$\underline{r}\times\underline{F} = \underline{r}\times m\underline{a} = \dot{\underline{H}}^O$$

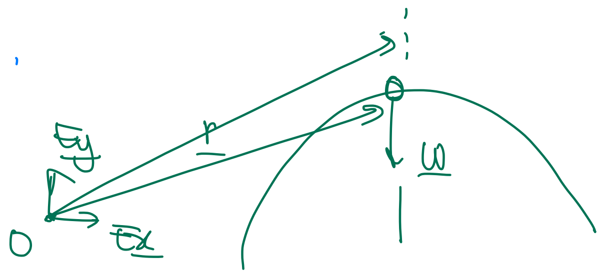
BoAM $\quad \underline{M}^O = \boxed{\underline{r}\times\underline{F} = \dot{\underline{H}}^O}$ The Balance of Angular Momentum for a particle.

For a particle, BoAM is derived from BoLM. This is not the case for systems of particles or rigid bodies, for which the BoAM is a law postulated by Euler known as Euler's IInd law.

$\boxed{\underline{M}^O = \underline{r}\times\underline{F}}$ is the sum of moments about pt O acting on a system.

To calculate the moment of a force about a certain point, you cross the position from that point to a point on the line of action of a force with the force.

eg.



$$\underline{M}^O = \underline{r} \times \underline{W} = (x\underline{E}_x + y\underline{E}_y) \times (-mg\underline{E}_y) = -mgx\underline{E}_z .$$

Conservation of Angular Momentum

$$\underline{M}^O = \underline{r} \times \underline{F} = \dot{\underline{H}}^O$$

integral form of BofAM
$$\boxed{\int_{t_A}^{t_B} \underline{M}^O dt = \underline{H}^O_B - \underline{H}^O_A}$$

In what case is $\underline{H}^O$ conserved?

$$\underline{H}^O \text{ conserved} \Rightarrow \dot{\underline{H}}^O = \underline{0} \Rightarrow \underline{M}^O .$$

In analogy to our previous developments, $\underline{H}^O$ might not be completely conserved, but it might be conserved in a certain direction.

$$\boxed{\underline{H}^O \cdot \underline{c} = \text{const}}$$

$$\frac{d}{dt}(\underline{H}^O \cdot \underline{c}) = \underline{0} = \dot{\underline{H}}^O \cdot \underline{c} + \underline{H}^O \cdot \dot{\underline{c}} = \boxed{\underline{M}^O \cdot \underline{c} + \underline{H}^O \cdot \dot{\underline{c}} = 0}$$

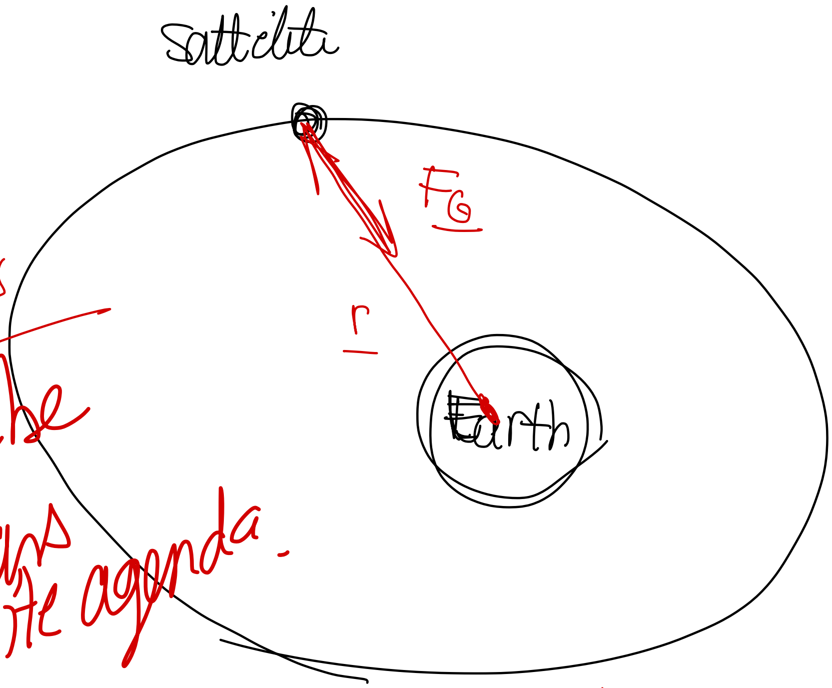
if $\underline{c}$ = const, then $\underline{H}^O \cdot \underline{c}$ = const $\Leftrightarrow$ $\boxed{\underline{M}^O \cdot \underline{c} = 0}$

if $\dot{x} = 0$, $x = c$; if $\dot{x} = c$, $x = ct$ if $\dot{\underline{H}}^O$ is const $\Rightarrow$ then $\underline{H}^O$ is not conserved.

Central force Problem $\underline{r} \parallel \underline{F}$

$$\underline{r} \times \underline{F} = \underline{0}$$

$$\Rightarrow \underline{H}^O \text{ is conserved.}$$



Kepler

Keplers law's

See youtube video on class website agenda.

$$\underline{r} \times m\underline{v} = r\underline{e_r} \times m(\dot{r}\underline{e_r} + r\dot{\theta}\underline{e_\theta})$$

$$= mr^2\dot{\theta}\, \underline{e_r} \times \underline{e_\theta}$$

$$= \boxed{mr^2\dot{\theta}\, \underline{E_z} = \text{const} = \underline{H}^O}$$

The energy of a system is conserved if the only forces doing work on the system are conservative.

$$T_B - T_A = W_{\underline{F_G},AB} = U_A - U_B$$

conservative

$$\boxed{T_B + U_B = T_A + U_A = E}$$

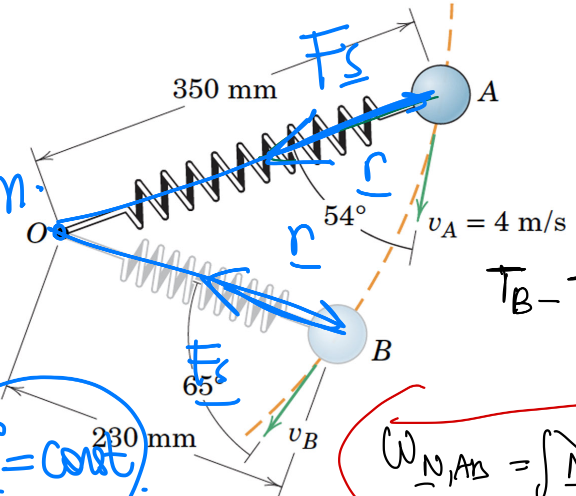
3/185 A particle of mass m moves with negligible friction on a horizontal surface and is connected to a light spring fastened at O. At position A the particle has the velocity $v_A = 4$ m/s. Determine the velocity v_B of the particle as it passes position B.

$\underline{r} \parallel \underline{F_s}$

$\Rightarrow$ central force problem.

$\underline{M}^O_{F_s} = \underline{r} \times \underline{F_s} = \underline{0}$

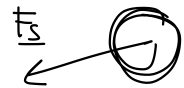
$\Rightarrow \underline{\dot{H}}^O = \underline{0}$, $\underline{H}^O = \text{const.}$



PROBLEM 3/185

$\odot \underline{N}$

$\otimes \underline{W}$



$T_B - T_A = \cancel{W_{\underline{N},AB}}^{\,0} + \cancel{W_{\underline{W},AB}}^{\,0} + W_{\underline{F_s},AB}$

conservative

$W_{\underline{N},AB} = \int \cancel{\underline{N} \cdot \underline{v}}_{\,0}\, dt$

All the forces doing work on the system are conserved, so the energy ~~of~~ the system is conserved.