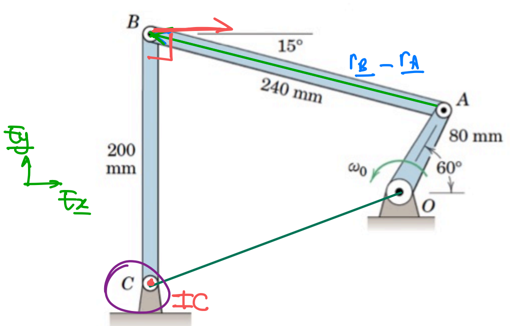
Today's Agenda Kinetics of RBs
* Translating motion
* Fixed point rotation
* General plane motion (time permitting).

set 12 p6

5/69 SS A four-bar linkage is shown in the figure (the ground "link" OC is considered the fourth bar). If the drive link OA has a counterclockwise angular velocity $\omega_0 = 10$ rad/s, determine the angular velocities of links AB and BC.



PROBLEM 5/69

① $\underline{v}_B - \underline{v}_A = \underline{\omega}_{AB} \times \overline{AB}\,(-\cos 15^\circ \underline{E}_x + \sin 15^\circ \underline{E}_y)$

$\underline{v}_A - \underline{v}_O = \underline{\omega}_{OA} \times \overline{OA}\,(\cos 60^\circ \underline{E}_x + \sin 60^\circ \underline{E}_y) \Rightarrow$ find $\underline{v}_A$

② $\underline{v}_B - \underbrace{\underline{v}_C}_{0} = \underline{\omega}_{BC} \times \overline{BC}\,\underline{E}_y = \omega_{BC}\underline{E}_z \times \overline{BC}\,\underline{E}_y = -\omega_{BC}\overline{BC}\,\underline{E}_x$

⚠ ~~$\underline{v}_B - \underline{v}_O$~~ belong to two ≠ bodies

$\underline{\omega}_{AB} = \omega_{AB}\underline{E}_z$

$\underline{\omega}_{BC} = \omega_{BC}\underline{E}_z$

①, ② $\Big\} \Longrightarrow$ 4 scalar equations.

$\left.\begin{array}{l}\underline{v}_B \quad 2 \\ \omega_{AB} \\ \omega_{BC}\end{array}\right\}$ 4 unknowns

Final Thursday Dec 19 @ 3:30. Nicely.

The Balance laws for a Rigid Body.

- BoLM (Euler I) $\underline{F} = m\underline{a}_c$

  $\underline{F} = \dot{\underline{G}}$, $\underline{G} = m\underline{v}_c$

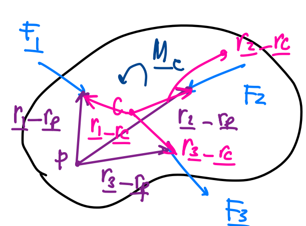
  $\underline{F}$ sum of the forces acting on the RBs.

- BoAM (Euler II)
  (a) $\underline{M}^O = \dot{\underline{H}}^O$ about a fixed pt O.
  (b) $\underline{M}^C = \dot{\underline{H}}^C$ about the center of mass C.
  (c) $\underline{M}^P = \dot{\underline{H}}^P + (\underline{v}_C - \underline{v}_P) \times \underline{G} = \dot{\underline{H}}^C + (\underline{r}_C - \underline{r}_P) \times m\underline{a}_C$ about any material point P on the RB.

These form of the BoAM are equivalent. We will show how we can obtain one from the other.

$\underline{M}^P$ sum of moments about point P.
= moments of forces + couples.

eg.



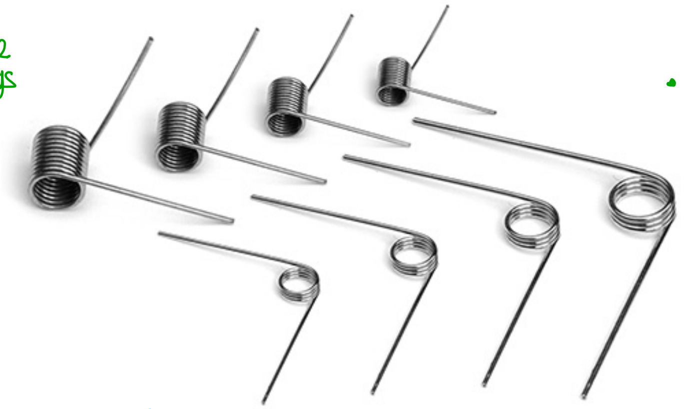
$\underline{F} = \underline{f_1} + \underline{f_2} + \underline{f_3}$

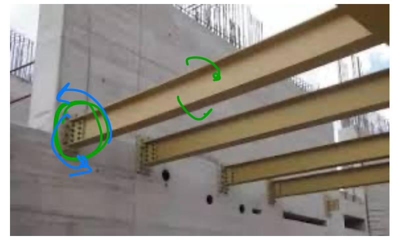
$\underline{M}^P = (\underline{r_1} - \underline{r_P}) \times \underline{f_1} + (\underline{r_2} - \underline{r_P}) \times \underline{f_2} + (\underline{r_3} - \underline{r_P}) \times \underline{f_3} + \underline{M_e}$

$\underline{M}^C = (\underline{r_1} - \underline{r_C}) \times \underline{f_1} + (\underline{r_2} - \underline{r_C}) \times \underline{f_2} + (\underline{r_3} - \underline{r_C}) \times \underline{f_3} + \underline{M_e}$.

What can provide couples:

- torsional springs





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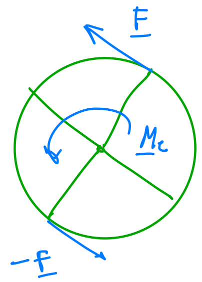
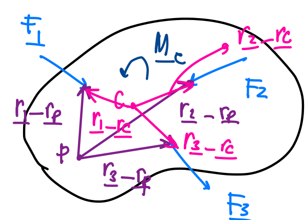
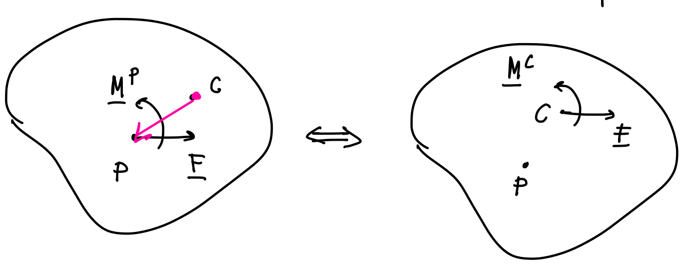


$\underline{M}^P = (\underline{r_1} - \underline{r_P}) \times \underline{f_1} + (\underline{r_2} - \underline{r_P}) \times \underline{f_2} + (\underline{r_3} - \underline{r_P}) \times \underline{f_3} + \underline{M_e}$

$\underline{M}^C = (\underline{r_1} - \underline{r_C}) \times \underline{f_1} + (\underline{r_2} - \underline{r_C}) \times \underline{f_2} + (\underline{r_3} - \underline{r_C}) \times \underline{f_3} + \underline{M_e}$.



Same system, I calculated $\underline{M}^P$ and $\underline{M}^C$. What is the relationship between $\underline{M}^C$ and $\underline{M}^P$?



★ $\underline{M}^C = \underline{M}^P + (\underline{r_P} - \underline{r_C}) \times \underline{F}$

I want to show that (a), (b), (c) are equivalent.

(a) $\underline{M}^O = \dot{\underline{H}}^O$ about a fixed pt O.

(b) $\underline{M}^C = \dot{\underline{H}}^C$ about the center of mass C.

(c) $\underline{M}^P = \dot{\underline{H}}^P + (\underline{v_C} - \underline{v_P}) \times \underline{G} = \underbrace{\dot{\underline{H}}^C \times (\underline{r_C} - \underline{r_P}) \times m\underline{a_C}}$ about any material point P on the RB.

Recall, $\boxed{\underline{H}^C = \underline{H}^P + (\underline{r_P} - \underline{r_C}) \times \underline{G}}$

Starting with $\underline{M}^C = \dot{\underline{H}}^C$, to obtain $\underline{M}^P = \dot{\underline{H}}^C + (\underline{r_C} - \underline{r_P}) \times m\underline{a_C}$

replace $\underline{M}^c = \underline{\dot H}^c$

$*\ \underline{M}^c = \underline{M}^P + (\underline{r}_P - \underline{r}_c)\times \underline{F}$

C: center of mass

P: any material pt on the RB.

$\underline{M}^P + (\underline{r}_P - \underline{r}_c)\times \underline{F} = \underline{\dot H}^c$

$\Longrightarrow \underline{M}^P = \underline{\dot H}^c + (\underline{r}_c - \underline{r}_P)\times \underline{F}$, $\underline{F} = m\underline{a}_c$ BoLM

$$\boxed{\underline{M}^P = \underline{\dot H}^c + (\underline{r}_c - \underline{r}_P)\times m\underline{a}_c}$$

$\underline{H}^c = \underline{H}^P + (\underline{r}_P - \underline{r}_c)\times \underline{G}$

$\underline{\dot H}^c = \underline{\dot H}^P + (\underline{v}_P - \underline{v}_c)\times \underline{G} + (\underline{r}_P - \underline{r}_c)\times \underline{\dot G}$

(c) $\boxed{\underline{M}^P = \underline{\dot H}^P + (\underline{v}_P - \underline{v}_c)\times \underline{G}} + \cancel{(\underline{r}_P - \underline{r}_c)\times \underline{\dot G}} + \cancel{(\underline{r}_c - \underline{r}_P)\times \underline{\dot G}}$

if P is a fixed point O on the body

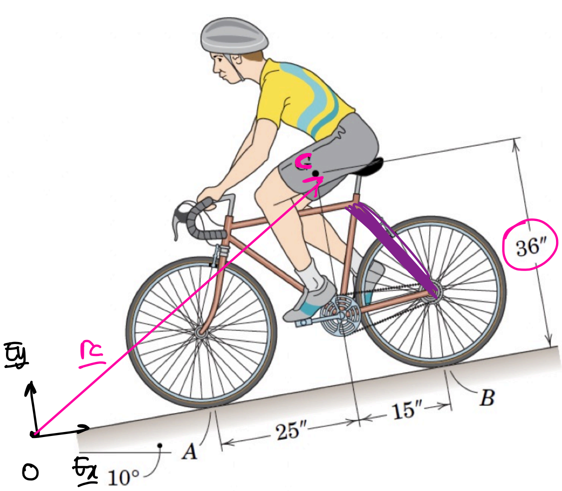
$\boxed{\underline{M}^O = \underline{\dot H}^O} + (\underbrace{\cancel{\underline{v}_O}}_{0} - \cancel{\underline{v}_c})\times \underbrace{\underline{G}}_{\cancel{m\underline{v}_c}}$ $\underline{0}$

(a)

let P be the center of mass C , $\boxed{\underline{M}^c = \underline{\dot H}^c} + \cancel{(\underline{v}_c - \underline{v}_c)}^{0}\times \underline{G}$

Example . set 6

6/12 The bicyclist applies the brakes as he descends the 10° incline. What deceleration a would cause the dangerous condition of tipping about the front wheel A? The combined center of mass of the rider and bicycle is at G.



1. $\underline{r}_c = x\underline{E}_x + \frac{36}{12}\underline{E}_y$ ft

$\underline{v}_c = \dot x \underline{E}_x$ ft/sec

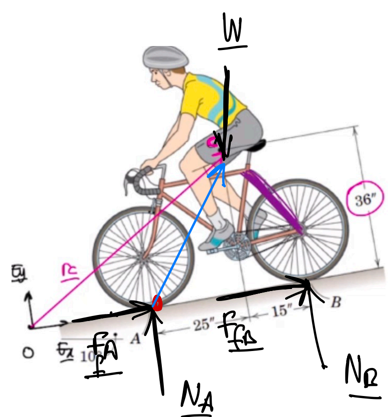
$\underline{a}_c = \ddot x \underline{E}_x$ ft/sec^2

$\underline{H}^c = \underline{I}^c\underline{\omega} = \underline{0}$, $\cancel{\underline{H}^c}^{0} = \underline{H}^P + (\underline{r}_P - \underline{r}_c)\times \underline{G}$

$\underline{H}^P = -(\underline{r}_P - \underline{r}_c)\times \underline{G}$

$= (\underline{r}_c - \underline{r}_P)\times m\underline{v}_c$

2. FBD.



$\underline{F}_B = \underline{0}$ lb.

$\underline{N}_B = \underline{0}$ lb

2. $\underline{F}_A = f_A\underline{E}_x$ lb

$\underline{N}_A = N_A\underline{E}_y$ lb

$\underline{W} = -mg(\cos 10°\,\underline{E}_x + \sin 10\,\underline{E}_y)$ lb

When the bike is about to tip about the front wheel, $\underline{N_B} = 0$ (since we are loosing contact at the rear wheel)
$\underline{F_B} = 0$

3. BoLM $\quad \underline{F} = m\underline{a}_c$

$$F_A \underline{E}_x + N_A \underline{E}_y - mg(\cos 10^\circ \underline{E}_x + \sin 10^\circ \underline{E}_y) = m\ddot{x}\underline{E}_x$$

BoAM about A. $\quad \underline{M}^A = \dot{\underline{H}}^A + (\underline{v}_A - \underline{v}_c) \times \underline{G} = \underbrace{\cancel{\dot{\underline{H}}^c}}_{0} + (\underline{r}_c - \underline{r}_A) \times m\underline{a}_c$

$$\underline{M}^A = (\underline{r}_c - \underline{r}_A) \times m\underline{a}_c$$

$m\ddot{x}\underline{E}_x \Rightarrow$ solve for $\ddot{x}$

$$\underline{M}^A = (\underline{r}_c - \underline{r}_A) \times \underline{W} = \left(\frac{25}{12}\underline{E}_x + \frac{26}{12}\underline{E}_y\right) \times \underline{W}$$

$$(\underline{r}_c - \underline{r}_A) =$$

$$\underbrace{(\underline{r}_A - \underline{r}_A)}_{0} \times \underline{N}_A$$

you could have done $\underline{M}^c = \dot{\underline{H}}^c$.

Curvilinear Motion.

Recall

$[\quad][\quad]$

- BoLM (Euler I) $\underline{F} = m\underline{a}_c$

  $\underline{F} = \dot{\underline{G}}$, $\underline{G} = m\underline{v}_c$

  $\underline{F}$ sum of the forces acting on the RBs.

- BoAM (Euler II)
  (a) $\underline{M}^o = \dot{\underline{H}}^o$ about a fixed pt O.
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  (c) $\underline{M}^P = \dot{\underline{H}}^P + (\underline{v}_c - \underline{v}_P) \times \underline{G} = \dot{\underline{H}}^c \times (\underline{r}_c - \underline{r}_P) \times m\underline{a}_c$ about any material point P on the RB.

$$\underline{H}^c = \left(I^c_{xx} \omega_x + I^c_{xy} \omega_y + I^c_{xz} \omega_z \right) \underline{e}_x + \left(I^c_{xy} \omega_x + I^c_{yy} \omega_y + I^c_{yz} \omega_z \right) \underline{e}_y + \left(I^c_{xz} \omega_x + I^c_{yz} \omega_y + I^c_{zz} \omega_z \right) \underline{e}_z .$$

FIXED

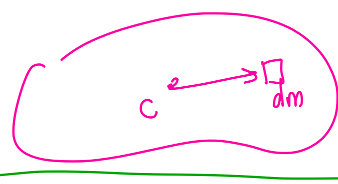
$\dot{\omega}_x$
$\dot{\omega}_y$
$\dot{\omega}_z$

$\dot{\underline{e}}_x = \underline{\omega} \times \underline{e}_x$
$\dot{\underline{e}}_y = \underline{\omega} \times \underline{e}_y$
$\dot{\underline{e}}_z = \underline{\omega} \times \underline{e}_z$

$$I^c_{xx} = \int (x^2 + y^2)\, dm$$

$$\underline{r} - \underline{r}_c = x\underline{e}_x + y\underline{e}_y + z\underline{e}_z$$

corotational derivative (derivative while keeping basis fixed) $\overset{o}{\underline{H}}{}^c$



$$\dot{\underline{H}}^c = \left(I^c_{xx} \dot{\omega}_x + I^c_{xy} \dot{\omega}_y + I^c_{xz} \dot{\omega}_z \right) \underline{e}_x + \left(I^c_{xy} \dot{\omega}_x + I^c_{yy} \dot{\omega}_y + I^c_{yz} \dot{\omega}_z \right) \underline{e}_y + \left(I^c_{xz} \dot{\omega}_x + I^c_{yz} \dot{\omega}_y + I^c_{zz} \dot{\omega}_z \right) \underline{e}_z + \underline{\omega} \times \underline{H}^c$$

Sidenote:

$\underline{r}_A - \underline{r}_B = x\underline{e}_x + y\underline{e}_y + z\underline{e}_z$

$\underline{v}_{rel} = \dot{x}\underline{e}_x + \dot{y}\underline{e}_y + \dot{z}\underline{e}_z = \overset{o}{\underline{r}}_{A/B}$

$\underline{a}_{rel} = \ddot{x}\underline{e}_x + \ddot{y}\underline{e}_y + \ddot{z}\underline{e}_z = \overset{oo}{\underline{r}}_{A/B}$

$$\boxed{\dot{\underline{H}}^c = \overset{o}{\underline{H}}{}^c + \underline{\omega} \times \underline{H}^c}$$

this simplifies $\omega_x = 0$, $\omega_y = 0$, $\underline{\omega} = \omega\, \underline{e}_z$

$$\dot{\underline{H}}^c = \left(I^c_{xz} \dot{\omega}_z \right) \underline{e}_x + \left(I^c_{yz} \dot{\omega}_z \right) \underline{e}_y + \left(I^c_{zz} \dot{\omega}_z \right) \underline{e}_z + \omega_z \underline{e}_z \times \left(I^c_{xz} \omega_z \underline{e}_x + I^c_{yz} \omega_z \underline{e}_y + I^c_{zz} \omega_z \underline{e}_z \right)$$

$w_z^2 I^C_{xz}\, e_y + I^C_{yz}\, w_z^2 (-e_x)$

$$\boxed{\dot{H}^C = (I^C_{xz}\dot{w}_z - I^C_{yz} w_z^2)\, e_x + (I^C_{yz}\dot{w}_z + I^C_{xz} w_z^2)\, e_y + I^C_{zz}\dot{w}_z\, e_z}$$

$\int_B zx\, dm$ $\quad\int yz\, dm$ $\quad\frac{1}{3}ml^2$

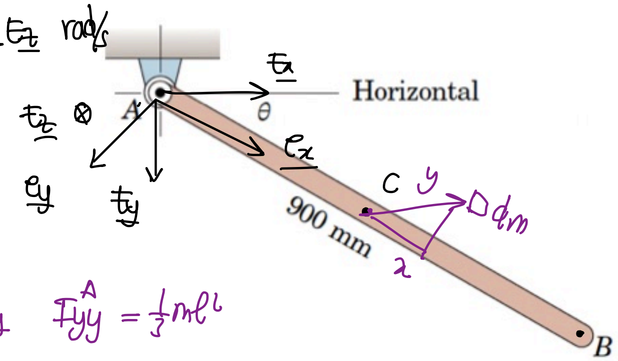
$$\dot{\mathbf{H}}^O = (\cancel{I^O_{xz}}\dot{\omega} - \cancel{I^O_{yz}}\omega^2)\mathbf{e}_x + (\cancel{I^O_{yz}}\dot{\omega} + \cancel{I^O_{xz}}\omega^2)\mathbf{e}_y + I^O_{zz}\dot{\omega}\mathbf{E}_z,$$
$$\dot{\mathbf{H}}^C = (I^C_{xz}\dot{\omega} - I^C_{yz}\omega^2)\mathbf{e}_x + (I^C_{yz}\dot{\omega} + I^C_{xz}\omega^2)\mathbf{e}_y + I^C_{zz}\dot{\omega}\mathbf{E}_z,$$
$$\dot{\mathbf{H}}^P = (I^P_{xz}\dot{\omega} - I^P_{yz}\omega^2)\mathbf{e}_x + (I^P_{yz}\dot{\omega} + I^P_{xz}\omega^2)\mathbf{e}_y + I^P_{zz}\dot{\omega}\mathbf{E}_z.$$

set17

6/37 The uniform slender bar AB has a mass of 8 kg and swings in a vertical plane about the pivot at A. If $\dot{\theta} = 2$ rad/s when $\theta = 30°$, compute the force supported by the pin at A at that instant.

$\underline{\omega}(\theta=30°) = 2\underline{E_z}$ rad/s

$g \downarrow$



PROBLEM 6/37

e_y $\quad I^A_{yy} = \frac{1}{3}ml^2$

$E_x \otimes$ $\quad I^A_{zz} = \frac{1}{3}ml^2$

1.

$\underline{e_x} = \cos\theta\, \underline{E_x} + \sin\theta\, \underline{E_y}$, $\quad \dot{\underline{e_x}} = \underline{\omega} \times \underline{e_x} = \dot{\theta}\underline{e_y}$

$\underline{e_y} = -\sin\theta\, \underline{E_x} + \cos\theta\, \underline{E_y}$, $\quad \dot{\underline{e_y}} = \underline{\omega} \times \underline{e_y} = -\dot{\theta}\underline{e_x}$

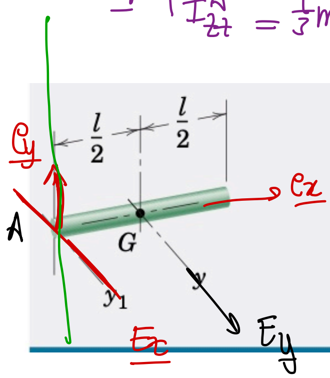
$\underline{r_c} = x\underline{e_x}$

$\underline{v_c} = x\dot{\underline{e_x}} = x\underline{\omega} \times \underline{e_x} = x\dot{\theta}\underline{e_y}$

$\underline{a_c} = x\ddot{\theta}\underline{e_y} - x\dot{\theta}^2\underline{e_x}$

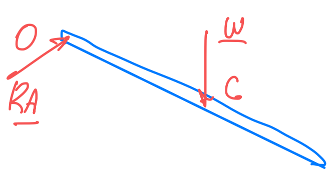
$\underline{H}^O = \underline{I}^O \underline{\omega}$

$\dot{\underline{H}}^O = I^O_{zz}\dot{w}\underline{E_z} = \frac{1}{3}ml^2\dot{w}\underline{E_z}.$



Uniform Slender Rod	—	$I_{yy} = \frac{1}{12}ml^2 \Leftarrow I^C_{zz}$ $I_{y_1y_1} = \frac{1}{3}ml^2$ $\quad I^A_{zz}$

2. FBD



$\underline{W} = -mg\underline{E_y}$

$\underline{R_A} = R_{Ax}\underline{E_x} + R_{Ay}\underline{E_y}$

3. BoLM $\underline{F} = m\underline{a}_c$

$$-mg\underline{E}_y + R_{Ax}\underline{E}_x + R_{Ay}\underline{E}_y = m(x\ddot{\theta}\underline{e}_y - x\dot{\theta}^2\underline{e}_x)$$

BoAM $\underline{M}^o = \underline{\dot{H}}^o$

$$(\underline{r}_c - \underline{r}_o) \times \underline{w} = \frac{1}{3}ml^2\dot{\omega}\underline{E}_z$$

$$x\underline{e}_x \times (-mg\underline{E}_y) = \frac{1}{3}ml^2\dot{\omega}\underline{E}_z$$

$$\underline{e}_x \times \underline{E}_y = \underline{e}_x \times (-\sin\theta\,\underline{e}_x + \cos\theta\,\underline{e}_y) = \cos\theta\,\underline{E}_z$$

$$-mgx\cos\theta\,\underline{E}_z = \frac{1}{3}ml^2\ddot{\theta}\,\underline{E}_z$$

$\cdot\underline{E}_z$

$$\boxed{\frac{1}{3}ml^2\ddot{\theta} + mgx\cos\theta = 0}$$

$x = \frac{l}{2}$

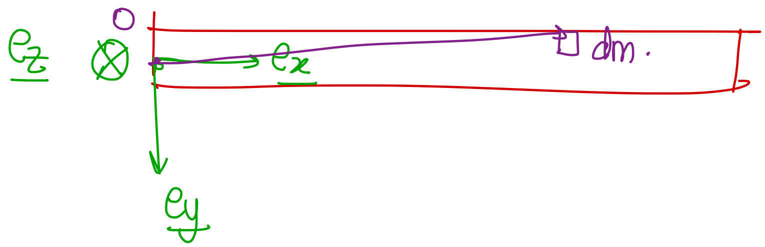
EOM

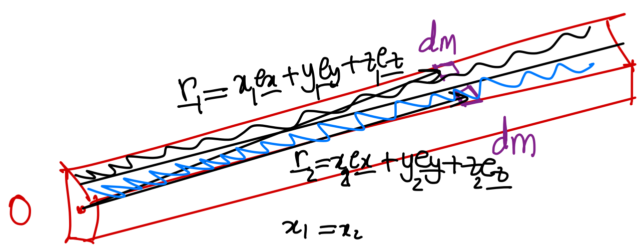
2nd

✓

$$\Rightarrow I^{O}_{xz} = \int_B x\cancel{z}\,dm = 0 \qquad I^{O}_{zz} = \int (x^2+y^2)\,dm$$

$$I^{O}_{yz} = \int_B y\cancel{z}\,dm. = 0$$





$x_1 = x_2$

$y_1 = y_2$

$z_1 = -z_2$

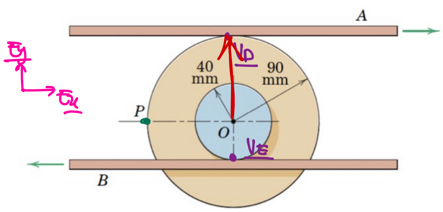
$dm\,\underline{r_1} + dm\,\underline{r_2}$

$\Rightarrow$ z components cancel out

$\int z\,dm = 0$

set 14

5/87 The attached wheels roll without slipping on the plates A and B, which are moving in opposite directions as shown. If $v_A = 60$ mm/s to the right and $v_B = 200$ mm/s to the left, determine the speeds of the center O and the point P for the position shown.



PROBLEM 5/87

$\vec{V}_A = 0.06\,\hat{e}_x$ m/s

$\vec{V}_B = -0.2\,\hat{e}_x$ m/s

V_o ?

V_P ?

$r_B = 0.04$ m

$r_A = 0.09$ m

$V_D = V_A$

$V_E = V_B$

$\vec{V}_D - \vec{V}_E = \vec{\omega} \times (\vec{r}_D - \vec{r}_E)$

$(0.06 + 0.2) =$

$\vec{V}_P - \vec{V}_o = \vec{\omega}_{OP} \times (\vec{r}_P - \vec{r}_o)$

$\vec{V}_P - \vec{V}_D = \dots$

$\vec{V}_D - \vec{V}_o = \vec{\omega}_{OD} \times (\vec{r}_D - \vec{r}_o)$

$V_o\,\hat{e}_x \qquad \omega_{OD}\,\hat{e}_z \qquad r_A\,\hat{e}_y$

2 eq. 2 unknowns.

$V_{rel} = V_{slip} = 0$

$V_P = V_{ground} = 0$

$V_{rel} = 0$

$V_E = V_{ground} = V_E = V_B$

$\vec{V}_E - \vec{V}_o = \vec{\omega}_{OE} \times (\vec{r}_E - \vec{r}_o)$

$V_o\,\hat{e}_x \qquad \omega_{OE}\,\hat{e}_z \qquad -r_B\,\hat{e}_y$

$0.06\,\hat{e}_x - V_o\,\hat{e}_x = -0.09\,\omega_{OD}\,\hat{e}_x$

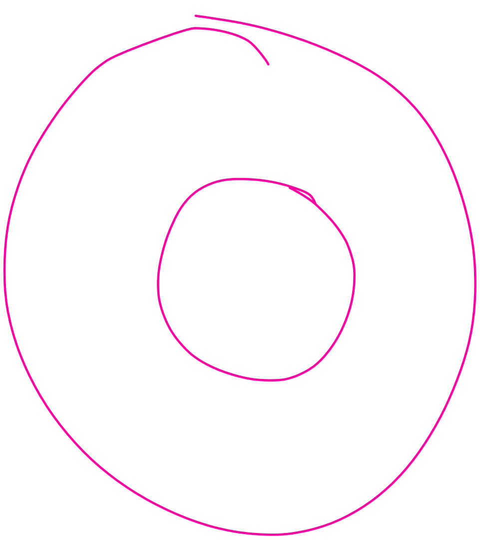
$0.06 - V_o = -0.09\,\omega$

$-0.2 - V_o = +0.04\,\omega$

if $\omega_{OD} = \omega_{OE}$, then

$$\frac{0.06 - V_o}{-0.2 - V_o} = \frac{-0.09}{+0.04}$$

$V_o = -120$ mm/s



120 mm/s