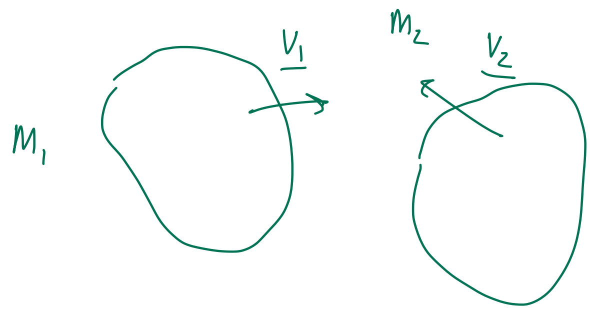
Announcement Midterm 1 will take place sometime next week.
It will cover the same material as previously announced (check website)
More details to come.

Today's lecture Collisions.

- deformation
  - < temporary
  - permanent.
- sound & heat will be generated.
- the velocities of the two bodies will change.

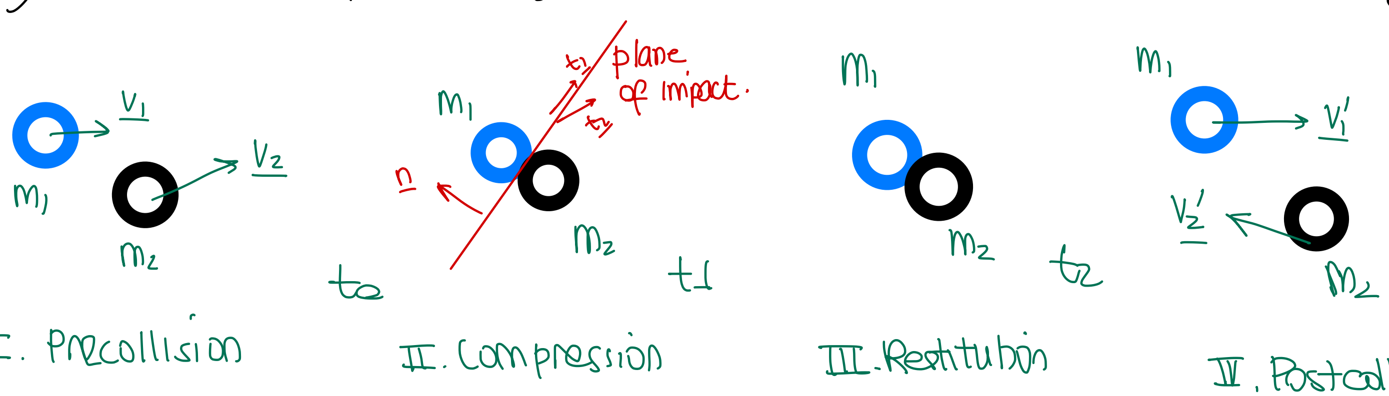


- some of the bodies might aquire or change angular velocity.

We are going to study a very simple model for collision.

Assumptions

1) assume that the colliding bodies can be modelled as particles.
2) we are neglecting rotations of the bodies. we assume that they are just translating.
3) all the particles on the body have the same velocity.
4) the impact is frictionless $\Rightarrow$ we will expand on this point.
5) Particles don't deform. We are going to define a parameter that quantifies deformation. e: coefficient of restitution.



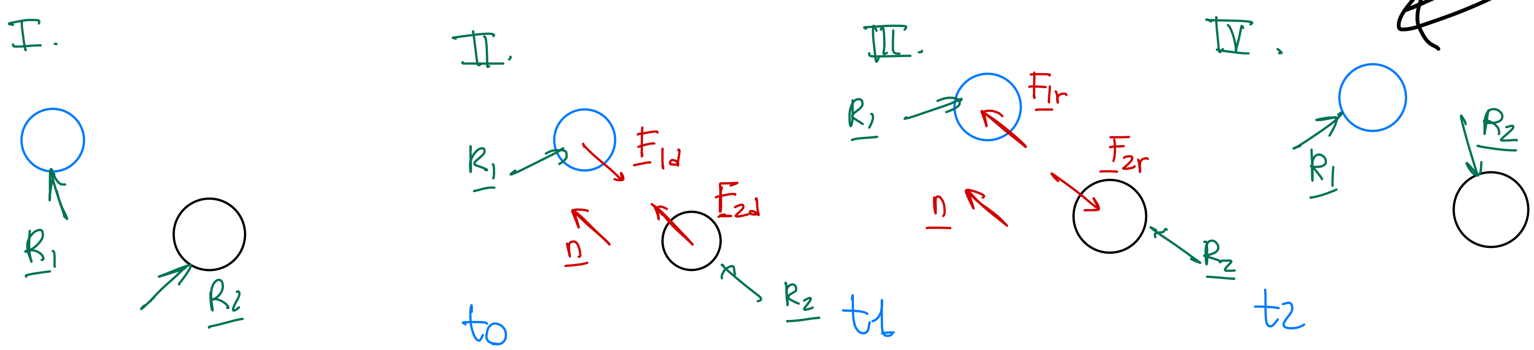
When the two particles touch there is no friction between them in the plane of impact.

all the impact forces are along $\underline{n}$.

t_0: Instant when particles first touch (beginning of II)
t_1: end on II, begining at III (instant at max compression)
t_2: when the particles are about to seperate (at end of III).

note at t_1, the particles have the same velocity in the n direction.

(⇔ at the instant of max deformation, the relative velocity of the particle in the normal direction is zero).



$\underline{R}_1$ and $\underline{R}_2$ are all the forces other than the collision forces (eg. they include $\underline{W}$, $\underline{N}$..)

$$\underline{F}_{1d} = f_{1d}\,\underline{n}$$
$$\underline{F}_{2d} = f_{2d}\,\underline{n}$$
deformation

$$\underline{F}_{1r} = f_{1r}\,\underline{n}$$
$$\underline{F}_{2r} = f_{2r}\,\underline{n}$$
restitution.

We are going to make two assumptions about the forces.

1. Linear impulse $t_0 < t < t_1$ of the forces acting on particle 1.

$$\int_{t_0}^{t_1} (\underline{R}_1 + \underline{F}_{1d})\,dt \approx \int_{t_0}^{t_1} \underline{F}_{1d}\,dt$$

very small time interval; smooth finite; large impulsive force

$t_1 < t < t_2$

$$\int_{t_1}^{t_2} (\underline{R}_1 + \underline{F}_{1r})\,dt \approx \int_{t_1}^{t_2} \underline{F}_{1r}\,dt$$

$$\int_{t_0}^{t_1} (\underline{R}_2 + \underline{f}_{2d})\,dt \approx \int_{t_0}^{t_1} \underline{f}_{2d}\,dt$$

$$\int_{t_1}^{t_2} (\underline{R}_2 + \underline{f}_{1r})\,dt \approx \int_{t_1}^{t_2} \underline{f}_{1r}\,dt$$

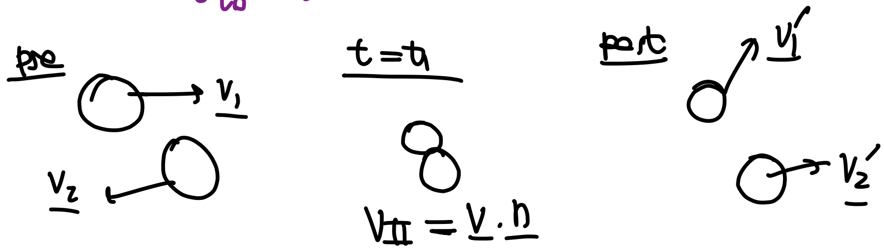
2. $\underline{F}_{1d} = -\underline{f}_{2d}$

$\underline{f}_{1r} = -\underline{F}_{2r}$

We now define the restitution coefficient e

$$\Longrightarrow e = \frac{\int_{t_1}^{t_2} \underline{F}_{1r}(\tau)\cdot\underline{n}\, d\tau}{\int_{t_0}^{t_1} \underline{F}_{1d}(\tau)\cdot\underline{n}\, d\tau} = \frac{\int_{t_1}^{t_2} \underline{F}_{2r}(\tau)\cdot\underline{n}\, d\tau}{\int_{t_0}^{t_1} \underline{F}_{2d}(\tau)\cdot\underline{n}\, d\tau}$$

we usually obtain this parameter experimentally



we are going to find a simpler expression for e in terms of pre-impact and post-impact velocities.

$$\int_{t_0}^{t_1} \underline{F}_{1d}(\tau)\cdot\underline{n}\, d\tau = m_1 V_{II} - m_1\, \underline{v}_1\cdot\underline{n}$$

$$\int_{t_1}^{t_2} \underline{F}_{1r}(\tau)\cdot\underline{n}\, d\tau = m_1(\underline{v}_1'\cdot\underline{n} - V_{II})$$

$$\int_{t_0}^{t_1} \underline{F}_{2d}(\tau)\cdot\underline{n}\, d\tau = m_2(V_{II} - \underline{v}_2\cdot\underline{n})$$

$$\int_{t_1}^{t_2} \underline{F}_{2r}(\tau)\cdot\underline{n}\, d\tau = m_2(\underline{v}_2'\cdot\underline{n} - V_{II})$$

$$\underline{F} = m\underline{a}$$

$$\underline{F} = \dot{\underline{G}}$$

$$\int_{t_A}^{t_B} \underline{F}\, dt = \underline{G}(t_B) - \underline{G}(t_A)$$

$$\int_{t_A}^{t_B} \underline{F}\cdot\underline{n}\, dt = \underline{G}_B\cdot\underline{n} - \underline{G}_A\cdot\underline{n}$$

for some fixed $\underline{n}$

$$e = \frac{\cancel{m_1}(\underline{V}_1'\cdot\underline{n} - V_{II})}{\cancel{m_1}(V_{II} - \underline{V}_1\cdot\underline{n})} = \frac{\cancel{m_2}(\underline{V}_2'\cdot\underline{n} - V_{II})}{\cancel{m_2}(V_{II} - \underline{V}_2\cdot\underline{n})}$$

We can manipulate these expressions to get rid of V_{II} and obtain

$$e = \frac{\underline{V}_2'\cdot\underline{n} - \underline{V}_1'\cdot\underline{n}}{\underline{V}_1\cdot\underline{n} - \underline{V}_2\cdot\underline{n}}$$

$$e\left(v_{II} - \underline{v}_1 \cdot \underline{n}\right) = \underline{v}_1' \cdot \underline{n} - v_{II}$$

$$v_{II}(1+e) = \underline{v}_1' \cdot \underline{n} + e\,\underline{v}_1 \cdot \underline{n}$$

$$v_{II} = \frac{\underline{v}_1' \cdot \underline{n} + e\,\underline{v}_1 \cdot \underline{n}}{1+e} = \frac{\underline{V}_2' \cdot \underline{n} + e\,\underline{V}_2 \cdot \underline{n}}{1+e}$$

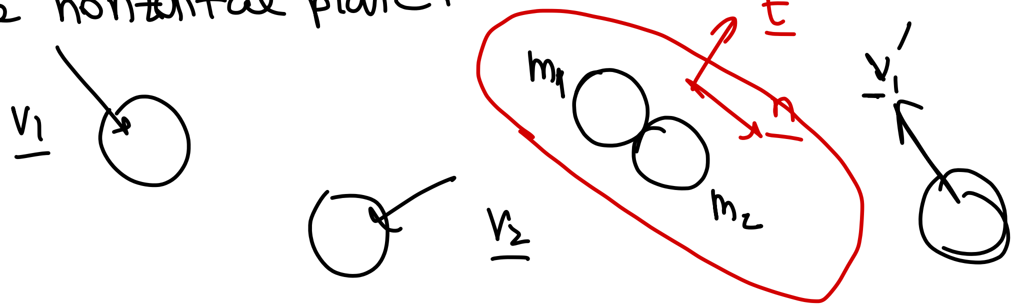
$$\underline{V}_1' \cdot \underline{n} + e\,\underline{V}_1 \cdot \underline{n} = \underline{V}_2' \cdot \underline{n} + e\,\underline{V}_2 \cdot \underline{n}$$

$$e\left(\underline{V}_1 \cdot \underline{n} - \underline{V}_2 \cdot \underline{n}\right) = \underline{V}_2' \cdot \underline{n} - \underline{V}_1' \cdot \underline{n}$$

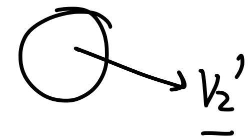
$$e = \frac{\underline{V}_2' \cdot \underline{n} - \underline{V}_1' \cdot \underline{n}}{\underline{V}_1 \cdot \underline{n} - \underline{V}_2 \cdot \underline{n}}.$$

Two particles of masses m_1 and m_2 have pre-impact velocities $\underline{v}_1$ and $\underline{v}_2$. Find the velocities of the particles post-impact $\underline{v}_1'$ & $\underline{v}_2'$.

In the horizontal plane:



think about conservations of linear momentum $\underline{G}$ in particular direction.



unknowns $\left.\begin{matrix}\underline{v}_1' \\ \underline{v}_2'\end{matrix}\right\} \Rightarrow \boxed{4}$ scalar unknowns.

equations $$e = \frac{\underline{v}_2' \cdot \underline{n} - \underline{v}_1' \cdot \underline{n}}{\underline{v}_1 \cdot \underline{n} - \underline{v}_2 \cdot \underline{n}} \quad (1)$$

$$\underline{F} = \dot{\underline{G}}$$

if $\underline{u}$ is a fixed direction,

if $\underline{F} \cdot \underline{u} = 0$

$\Rightarrow \dot{\underline{G}} \cdot \underline{u} = 0$

$\underline{G} \cdot \underline{u} = \text{const} = \text{conserved}$

consider the combined system $m_1 + m_2$ during collision. The linear momentum of the whole system is conserved. (The collision forces are internal to the system)

$$\underline{F} = \underline{0} \qquad \underline{f}_1 + \underline{f}_2 = \underline{0}$$

$$\underline{f}_1 = \dot{\underline{G}}_1, \quad \underline{f}_2 = \dot{\underline{G}}_2$$

$$\dot{\underline{G}} = \underline{0}$$

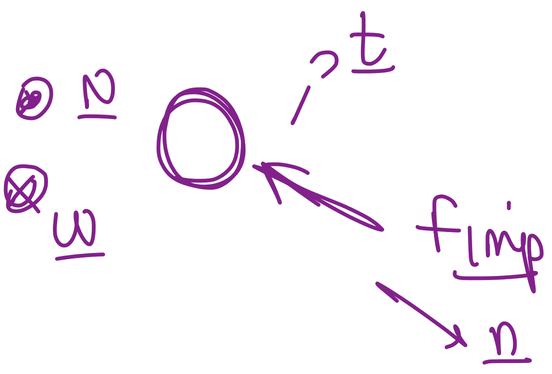
$$\dot{\underline{G}}_1 + \dot{\underline{G}}_2 = \underline{0}$$

$$\underline{G}_1 + \underline{G}_2 = \text{const}$$

$$\boxed{m_1 \underline{v}_1 + m_2 \underline{v}_2 = m_1 \underline{v}_1' + m_2 \underline{v}_2'} \quad (2) \quad (3)$$

$$(\underline{G}_1 + \underline{G}_2)_{pre} = (\underline{G}_1 + \underline{G}_2)_{post}$$

if you look at particle 1 alone.



$(\underline{f}_{1d}, \underline{f}_{1r})$

$$\underline{f_1}\cdot\underline{t} = 0$$

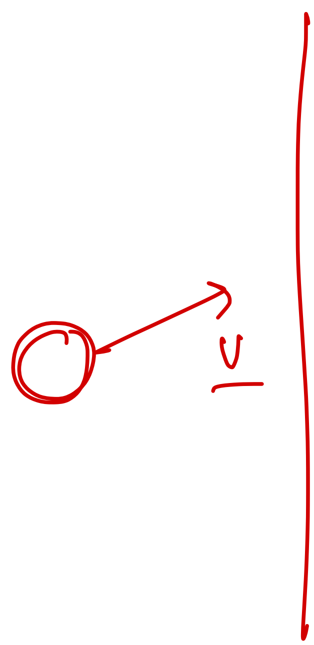
$$\dot{\underline{G_1}}\cdot\underline{t} = 0$$

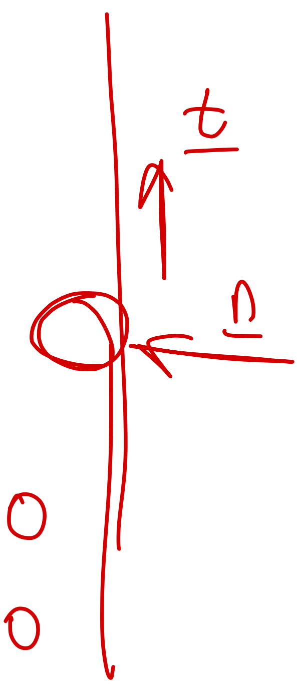
the linear momentum of particle 1 in the $\underline{t}$ direction is conserved

$$\boxed{\underline{G_1}\cdot\underline{t} = \text{const}}$$

$$m_1\underline{v_1}\cdot\underline{t} = m_1\underline{v_1'}\cdot\underline{t} \qquad (4)$$

eg.





$$\underline{f}\cdot\underline{t} = 0$$

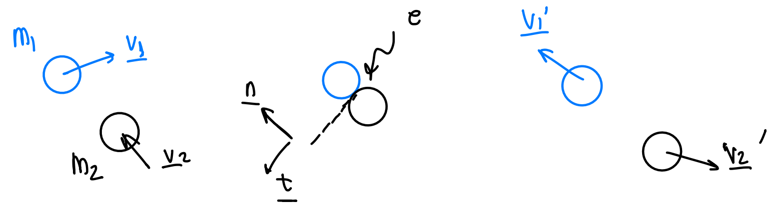
$$\dot{\underline{G}}\cdot\underline{t} = 0$$

$$m\underline{v}\cdot\underline{t} = m\underline{v_1'}\cdot\underline{t}$$

the component of the velocity tangent to all will stay the same.

Consider two particles of masses m_1 and m_2 with pre-impact velocities $\underline{v_1}$ and $\underline{v_2}$. The coefficient of restitution of the impact between both particles is e. Calculate the post impact velocities of the particles.

This is happening in the horizontal plane. No friction.



- we want to determine $\underline{v_1}', \underline{v_2}'$ $\Rightarrow$ 4 scalar unknowns.
- We need 4 equations to solve for these unknowns.

① $\Rightarrow$ $$\boxed{e = \frac{\underline{v_2}'\cdot\underline{n} - \underline{v_1}'\cdot\underline{n}}{\underline{v_1}\cdot\underline{n} - \underline{v_2}\cdot\underline{n}}}$$

②+③ conservation of linear momentum of the combined system.

$\underline{F_1} + \underline{F_2} = \underline{0}$ $\quad$ $\underline{F_1}$: external force on particle 1

$\dot{\underline{G_1}} + \dot{\underline{G_2}} = \underline{0}$ $\quad$ $\underline{F_2}$: external force on particle 2.

$\underline{G_1} + \underline{G_2}$ = conserved = constant

$\Rightarrow$ $\boxed{m_1 \underline{v_1}' + m_2 \underline{v_2}' = m_1 \underline{v_1} + m_2 \underline{v_2}}$ $\quad$ $* \cdot \underline{t}$ are not idependent.

④ conservation of linear momentum of m_1 along $\underline{t}$ (all impact forces are along $\underline{n}$)

$\underline{F_1}\cdot\underline{t} = 0$

$\dot{\underline{G_1}}\cdot\underline{t} = 0$

$\underline{G_1}\cdot\underline{t} = \text{const}$

$\Rightarrow$ $\boxed{m_1 \underline{v_1}'\cdot\underline{t} = m_1 \underline{v_1}\cdot\underline{t}}$ $*$

$$\boxed{⑤ + ④ \Longleftrightarrow (2+3)\cdot\underline{t}}$$

$\Rightarrow$ ⑤ conservation of linear momentum of m_2 along $\underline{t}$ (all impact forces are along $\underline{n}$)

$\underline{F_2}\cdot\underline{t} = 0$

$\dot{\underline{G_2}}\cdot\underline{t} = 0$

$\underline{G_2}\cdot\underline{t} = \text{const}$

$\Rightarrow$ $\boxed{m_1 \underline{v_2}'\cdot\underline{t} = m_1 \underline{v_2}\cdot\underline{t}}$ $*$

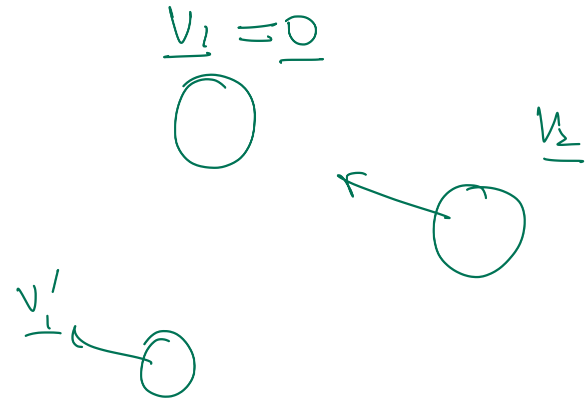
Velocities along tangent direction are conserved

$$T_1' - T_1 = \cancel{U_{\underline{W_1}}}_{0} + \cancel{U_{\underline{N_1}}}_{0} + U_{\underline{F_{1d}}} + U_{\underline{F_{1r}}}$$

$$\frac{1}{2} m_1 (v_1')^2 - \frac{1}{2} m (v_1)^2 = U_{\underline{F_{1d}}} + U_{\underline{F_{1r}}}$$

$$T_2' - T_2 = \cancel{U_{\underline{W_1}}}_{0} + \cancel{U_{\underline{N_2}}}_{0} + U_{\underline{F_{2d}}} + U_{\underline{F_{2r}}}$$

$$T_1' + T_2' - (T_1 + T_2) = \underbrace{U_{\underline{F_{1d}}} + U_{\underline{F_{1r}}} + U_{\underline{F_{2d}}} + U_{\underline{F_{2r}}}}$$



$$T_1' + T_2' - (T_1 + T_2)$$

$$= \frac{1}{2} m(v_1')^2 + \frac{1}{2} m(v_2')^2 - \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$

$$= \underbrace{\frac{m_1 m_2}{2(m_1+m_2)} (\underline{v_1} \cdot \underline{n} - \underline{v_2} \cdot \underline{n})^2}_{>0} \underbrace{(e^2 - 1)}_{=0 \text{ if } e=1}.$$

if $e = 1$, $T_1' + T_2' = T_1 + T_2$

kinetic energy is conserved

perfectly elastic collision.

if $0 < e < 1$ $\quad T_1' + T_2' - (T_1 + T_2) < 0$.

$$T_1' + T_2' < T_1 + T_2$$

inelastic collision.

if $e = 0 \rightarrow$ max KE loss, plastic collision.

4. [03-201]

for the system of the ball, $\underline{F}\cdot\underline{n} \neq 0$, $\underline{F}\cdot\underline{t} = 0$

$\underline{G}\cdot\underline{t} = \text{const.}$

3/201 Determine the value of the coefficient of restitution e for which the outgoing angle is one-half of the incoming angle θ as shown. Evaluate your general expression for $\theta = 40°$.

$\underline{v} = v(\cos\theta\,\underline{E_x} - \sin\theta\,\underline{E_y})$

$\underline{v}' = v'(\cos\frac{\theta}{2}\underline{E_x} + \sin\frac{\theta}{2}\underline{E_y})$

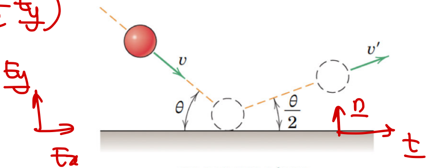
$\underline{v}\cdot\underline{E_x} = \underline{v}'\cdot\underline{E_x}$

$$\boxed{v\cos\theta = v'\cos\frac{\theta}{2}} \quad ①$$

$\theta = 40°$

unknowns v, v'

$$\frac{v'}{v} = \frac{\cos\theta}{\cos\frac{\theta}{2}}$$



PROBLEM 3/201

in the normal direction

$$e = \frac{\cancel{v_2'} \cdot n - v_1' \cdot n}{v_1 \cdot n - \cancel{v_2} \cdot n}$$

1. ball
2. wall $v_2 = ?$

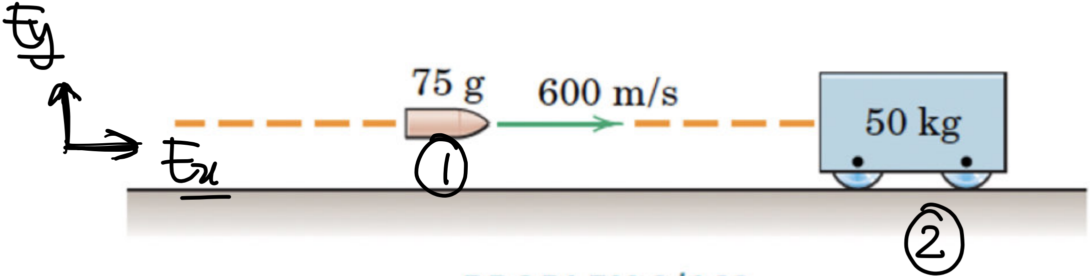
$$\boxed{e = -\frac{v' \sin\left(\frac{\theta}{2}\right)}{-v \sin\theta}}$$ (2)

$$e = \frac{v'}{v} \frac{\sin\left(\frac{\theta}{2}\right)}{\sin\theta}$$

$$\boxed{e = \frac{\cos(\theta)}{\cos\left(\frac{\theta}{2}\right)} \frac{\sin\left(\frac{\theta}{2}\right)}{\sin(\theta)}}$$

What happens to the total energy of the system during collision?

3/142 A 75-g projectile traveling at 600 m/s strikes and becomes embedded in the 50-kg block, which is initially stationary. Compute the energy lost during the impact. Express your answer as an absolute value $|\Delta E|$ and as a percentage n of the original system energy E.



PROBLEM 3/142

$\underline{v_1} = 600\,\underline{E_x}$ m/s

$\underline{v_2} = \underline{0}$ m/s

$\underline{v_1'} = ?$

$\underline{v_2'} = ?$

$\underline{v_1'} = \underline{v_2'} = \underline{v'}$ because particle become embedded

$\underline{v_1'} \cdot \underline{E_y} = 0$

$\underline{v_2'} \cdot \underline{E_y} = 0$

} because of conservation of linear momentum of each body along $\underline{E_y}$ (tangent to impact)

the linear momentum of the combined system is conserved.

$$m_1 \underline{v_1} + m_2 \cancel{\underline{v_2}} = (m_1 + m_2)\, \underline{v'}$$