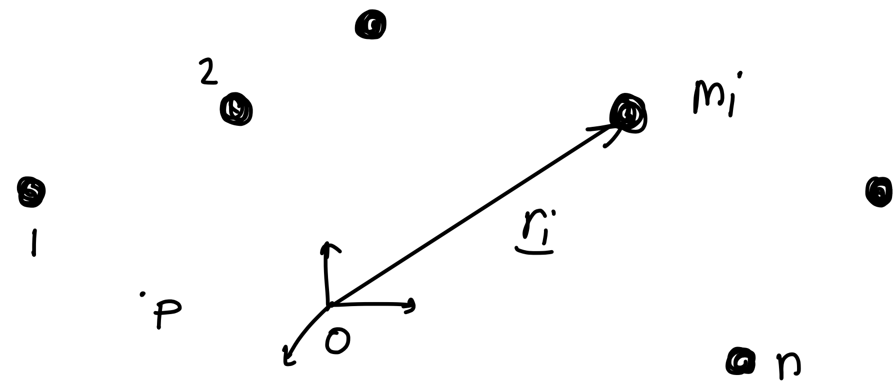
Announcements

- Midterm 1 happening soon, prepare for it. More details to follow (today or tomorrow)
- HW5 due Today
- HW6 assigned today due next week.
- Hybrid learning!

~~Dynamics~~ of Systems of Particles

Kinematics



Consider a system of n particles. A typical particle has

- mass m_i
- position vector with respect to O $\underline{r_i}$
- velocity vector $\underline{v_i} = \dfrac{d\underline{r_i}}{dt}$
- acceleration vector $\underline{a_i} = \dfrac{d^2 \underline{r_i}}{dt^2} = \dfrac{d\underline{v_i}}{dt}$.
- linear momentum $\underline{G_i} = m_i \underline{v_i}$
- angular momentum relative to some point P. $\underline{H}_{P_i} = (\underline{r_i} - \underline{r_P}) \times \underline{G_i}$.
- kinetic energy $T_i = \frac{1}{2} m_i \underline{v_i} \cdot \underline{v_i}$.

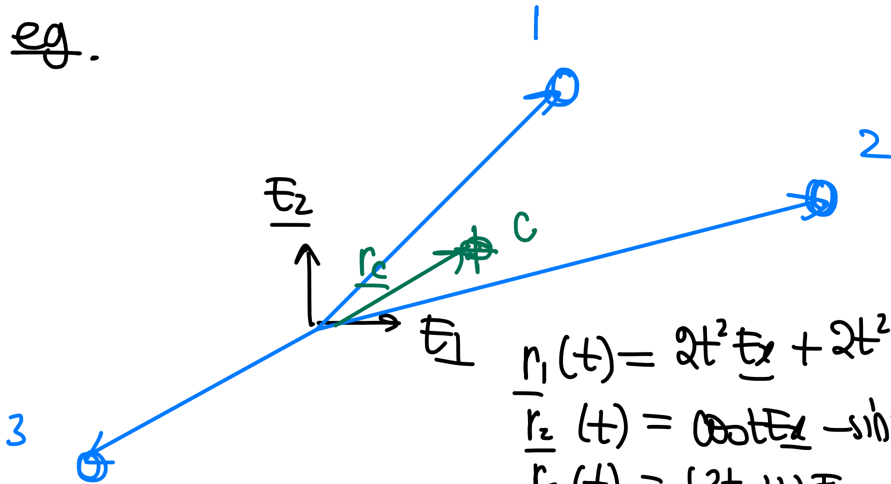
Total mass of the system of particles $m = \sum_{i=1}^{n} m_i$

$= m_1 + m_2 + \dots + m_n$

Define the center of mass of system C to have position vector

$$\underline{r_c} = \frac{\sum_{i=1}^{n} m_i \underline{r_i}}{m} = \sum_{i=1}^{n} \frac{m_i}{m} \underline{r_i}$$

weighted sum of the position vectors (average) eg.



$\underline{r_1}(t) = 2t^2\underline{E_x} + 2t^2\underline{E_y}$

$\underline{r_2}(t) = \cos t\,\underline{E_x} - \sin t\,\underline{E_y}$

$\underline{r_3}(t) = (3t+1)\underline{E_x}$

$m_1 = 2$ kg

$m_2 = 4$ kg

$m_3 = 1$ kg

$\underline{r_1} = 2\underline{E_x} + 2\underline{E_y}$ m

$\underline{r_2} = 4\underline{E_x} + 1\underline{E_y}$ m

$\underline{r_3} = -\underline{E_x} - \underline{E_y}$ m

$$m = \sum_{i=1}^{m} m_i = m_1 + m_2 + m_3 = 7.$$

$$\underline{r_c} = \frac{\sum_{i=1}^{n} m_i \underline{r_i}}{m}$$

$$= \frac{m_1\underline{r_1} + m_2\underline{r_2} + m_3\underline{r_3}}{m}$$

$$= \frac{2(2\underline{E_x} + 2\underline{E_y}) + 4(4\underline{E_x} + 1\underline{E_y}) + 1(-\underline{E_x} - \underline{E_y})}{7}$$

$$= \frac{19\underline{E_x} - 7\underline{E_y}}{7}$$

(derivative of a sum is the sum of derivatives)

$$\underline{r_c} = \frac{\sum m_i \underline{r_i}}{m},$$

$$\underline{v_c} = \frac{d}{dt}\left(\frac{\sum m_i \underline{r_i}}{m}\right) = \frac{\sum m_i \frac{d\underline{r_i}}{dt}}{m} = \frac{\sum_{i=1}^{n} m_i \underline{v_i}}{m},$$

$$\underline{a_c} = \frac{d}{dt}\underline{v_c} = \dots = \frac{\sum_{i=1}^{n} m_i \underline{a_i}}{m}.$$

$$\underline{V_c} = \frac{\sum_{i=1}^{n} m_i \underline{v_i}}{m}$$

$$\sum_{i=1}^{n} m_i \underline{v_i} = m\,\underline{v_c}$$

$\underline{G_i} = m_i \underline{v_i}$

The linear momentum of a system is defined to be equal to the sum of the linear momenta of the constituents

$$\underline{G} = \sum_{i=1}^{n} m_i \underline{v_i} = m \underline{v_c}$$

$\underline{G}$ is also equal to the linear momentum of the center of mass c.

$$\underline{r_c} = \frac{\sum m_i \underline{r_i}}{m}$$

eg. $(1+2+3)(4)$

$= (1)(4) + (2)(4) + (3)(4)$

$$\Rightarrow \quad m \underline{r_c} = \sum_{i=1}^{n} m_i \underline{r_i}$$

$$\sum_{i=1}^{n} (m_i \underline{r_i}) - m \underline{r_c} = 0$$

$$\sum_{i=1}^{n} (m_i \underline{r_i}) - \underbrace{\sum_{i=1}^{n} m_i \underline{r_c}}_{} = \underline{0}$$

$$= \left(\sum_{i=1}^{n} m_i\right) \underline{r_c} = \sum_{i=1}^{n} (m_i \underline{r_c}) \Leftarrow$$

$$\sum_{i=1}^{n} (m_i \underline{r_i} - m_i \underline{r_c}) = 0$$

$$\boxed{\sum_{i=1}^{n} m_i (\underline{r_i} - \underline{r_c}) = 0} \Leftarrow$$

$$\Leftrightarrow \sum_{k=1}^{n} m_k (\underline{r_k} - \underline{r_c}) = 0$$

$$\underline{v_c} = \frac{\sum m_i \underline{v_i}}{m}$$

$$m\underline{v_c} = \sum_{i=1}^{n} m_i \underline{v_i}$$

$$\sum_{i=1}^{n} \left(m_i \underline{v_i}\right) - m\underline{v_c} = 0$$

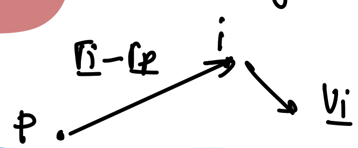
$$\boxed{\sum_{i=1}^{n} m_i(\underline{v_i} - \underline{v_c}) = 0} \Leftarrow$$

The angular momentum of a system of particles is the sum of the angular momenta of its constituents.

$$\underline{H}^{P_i} = (\underline{r_i} - \underline{r_P}) \times m_i \underline{v_i}$$

angular momentum of particle i about pt P.

$$\underline{H}^{C_i} = (\underline{r_i} - \underline{r_c}) \times m_i \underline{v_i}$$



$$\underline{H}^{P} = \sum_{i=1}^{n} (\underline{r_i} - \underline{r_P}) \times m_i \underline{v_i} = \underline{H}^{c} + (\underline{r_c} - \underline{r_P}) \times \underline{G}$$

where $\underline{H}^{c} = \sum_{k=1}^{n} (\underline{r_k} - \underline{r_c}) \times m_k \underline{v_k} = \sum_{k=1}^{n} (\underline{r_k} - \underline{r_c}) \times m_k (\underline{v_k} - \underline{v_c})$.

angular momentum about the CG.

derivation

$$\underline{H}^{P} = \sum_{i=1}^{n} (\underline{r_i} - \underline{r_P}) \times m_i \underline{v_i}$$

$$= \sum_{i=1}^{n} (\underline{r_i} + \underline{r_c} - \underline{r_c} - \underline{r_P}) \times m_i \underline{v_i}$$

$$= \sum_{i=1}^{n} (\underline{r_i} - \underline{r_c}) \times m_i \underline{v_i} + \sum_{i=1}^{n} (\underline{r_c} - \underline{r_P}) \times m_i \underline{v_i}$$

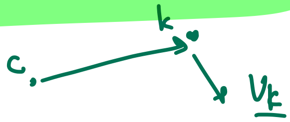
$$= \underline{H}^{c} + \sum_{i=1}^{n} (\underline{r_c} - \underline{r_P}) \times m_i \underline{v_i}$$

$\underline{r_i}$: position vector of particle i

$\underline{r_P}$: position vector of some random pt P.

$\underline{r_c}$: position vector of CG

$$\underline{H}^c = \sum_{k=1}^{n} (\underline{r}_k - \underline{r}_c) \times m_k \underline{v}_k$$



$$= \sum_{k=1}^{n} (r_k - r_c) \times m_k (\underline{v}_k - \underline{v}_c + \underline{v}_c)$$

$$= \sum_{k=1}^{n} (\underline{r}_k - \underline{r}_c) \times m_k (\underline{v}_k - \underline{v}_c) + \underbrace{\sum_{k=1}^{n} (\underline{r}_k - \underline{r}_c) \times m_k \underline{v}_c}_{\underline{0}}$$

$$\int 2x^2 dx = 2\int x^2 dx$$

$$\Downarrow$$

$$= \underline{v}_c \underbrace{\sum_{k=1}^{n} m_k (\underline{r}_k - \underline{r}_c)}_{\underline{0}}$$

Kinetic Energy

The kinetic energy of a system is defined to be equal to the sum of the kinetic energy of the constituents. ⚠

① $T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \frac{1}{2} m_i \underline{v}_i \cdot \underline{v}_i \neq \cancel{\frac{1}{2} m \underline{v}_c \cdot \underline{v}_c}$.

In terms of the center of mass

② $T = \frac{1}{2} m \underline{v}_c \cdot \underline{v}_c + \frac{1}{2} \sum_{k=1}^{n} m_k (\underline{v}_k - \underline{v}_c) \cdot (\underline{v}_k - \underline{v}_c)$.

Derivation to get ② from ①.

$$T = \sum_{i=1}^{n} \frac{1}{2} m_i \underline{v}_i \cdot \underline{v}_i$$

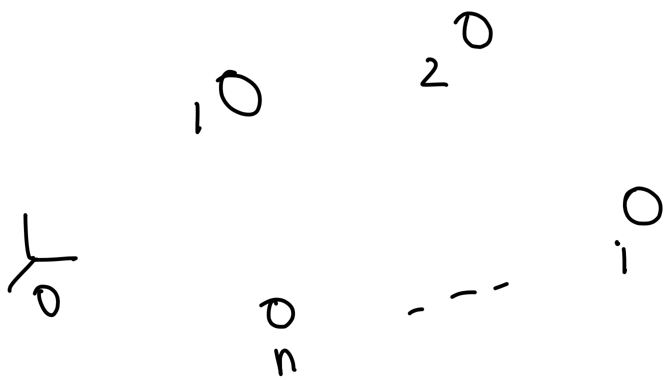
$$= \sum_{i=1}^{n} \frac{1}{2} m_i (\underline{v}_i - \underline{v}_c + \underline{v}_c) \cdot (\underline{v}_i - \underline{v}_c + \underline{v}_c)$$

$$= \sum_{i=1}^{n} \frac{1}{2} m_i (\underline{v}_i - \underline{v}_c) \cdot (\underline{v}_i - \underline{v}_c) + \sum_{i=1}^{n} \frac{1}{2} m_i \underline{v}_c \cdot \underline{v}_c + \sum_{i=1}^{n} 2 \cdot \frac{1}{2} m_i (\underline{v}_i - \underline{v}_c) \cdot \underline{v}_c$$

$$= \sum_{i=1}^{n} \frac{1}{2} m_i (\underline{v}_i - \underline{v}_c) \cdot (\underline{v}_i - \underline{v}_c) + \frac{1}{2} \underbrace{\left(\sum_{i=1}^{n} m_i\right)}_{m} \underline{v}_c \cdot \underline{v}_c + \sum_{i=1}^{n} \cancel{2} \cdot \cancel{\frac{1}{2}} m_i (\underline{v}_i - \underline{v}_c) \cdot \underline{v}_c$$

$$\underbrace{\left[\sum_{i=1}^{n} m_i(\underline{v_i} - \underline{v_c})\right]}_{0} \cdot \underline{v_c}$$

Kinetics of a system of particles : linear momentum.



For each individual particle $\underline{F_i} = m_i \underline{a_i}$.

For the system of particles $\boxed{\underline{F} = m \underline{a_c}}$

$$\sum \underline{F_i} = \underbrace{\sum_{i=1}^{n} m_i \underline{a_i}}$$

$$\equiv \underline{F}$$

by definition.

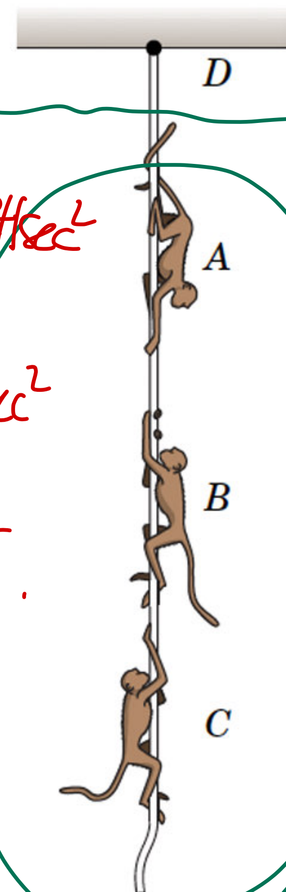
4/8 Three monkeys A, B, and C weighing 20, 25, and 15 lb, respectively, are climbing up and down the rope suspended from D. At the instant represented, A is descending the rope with an acceleration of 5 ft/sec^2, and C is pulling himself up with an acceleration of 3 ft/sec^2. Monkey B is climbing up with a constant speed of 2 ft/sec. Treat the rope and monkeys as a complete system and calculate the tension T in the rope at D.

1

$\underline{a}_A = -5\,\underline{e}_y$ ft/sec^2

$\underline{a}_B = 3\,\underline{e}_y$ ft/sec^2

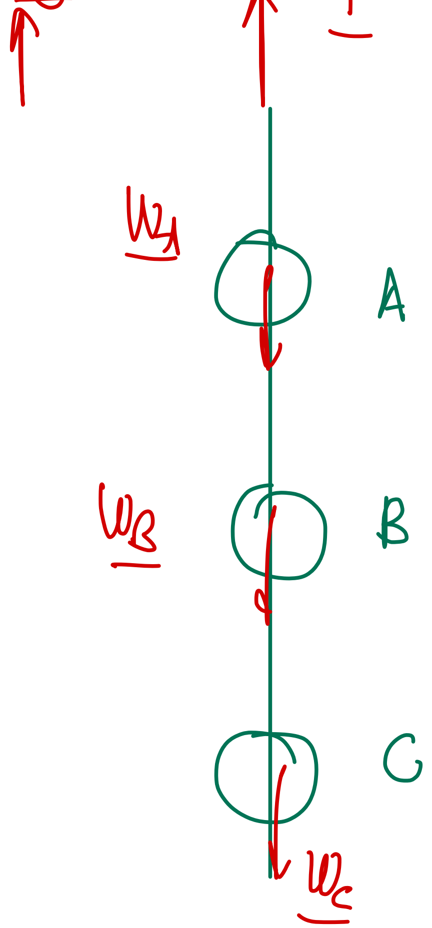
$\underline{a}_C = \underline{0}$ ft/sec^2



our system.

PROBLEM 4/8

(2)



$$\underline{T} = T\underline{E}_y$$

$$\underline{W}_A = -m_A g \underline{E}_y$$

$$\underline{W}_B = -m_B g \underline{E}_y$$

$$\underline{W}_C = -m_C g \underline{E}_y$$

(3)

$$\underline{F} = m\underline{a}_C = \sum_{i=1}^{3} m_i \underline{a}_i$$

$$\boxed{T\underline{E}_y - m_A g \underline{E}_y - m_B g \underline{E}_y - m_C g \underline{E}_y = m_A \underline{a}_A + m_B \underline{a}_B + m_C \underline{a}_C}$$

(4) $\underline{E}_y$ $\quad T = \ldots$