MECH230 - Fall 2024
Recommended Problems - Set 01

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Kinematics in Cartesian Coordinates The position, velocity, and acceleration vector of a particle expressed in Cartesian coordinates are respectively

$$\begin{aligned} \mathbf{r} &= x\mathbf{E}_x + y\mathbf{E}_y + z\mathbf{E}_z, \\ \mathbf{v} &= v_x\mathbf{E}_x + v_y\mathbf{E}_y + v_z\mathbf{E}_z = \dot{x}\mathbf{E}_x + \dot{y}\mathbf{E}_y + \dot{z}\mathbf{E}_z, \\ \mathbf{a} &= a_x\mathbf{E}_x + a_y\mathbf{E}_y + a_z\mathbf{E}_z = \ddot{x}\mathbf{E}_x + \ddot{y}\mathbf{E}_y + \ddot{z}\mathbf{E}_z. \end{aligned} \tag{1}$$

Rectilinear Motion Consider a rectilinear motion of a particle in the direction of $\mathbf{E}_x$.

$$\begin{aligned} \mathbf{r} &= x\mathbf{E}_x, \\ \mathbf{v} &= v\mathbf{E}_x = \dot{x}\mathbf{E}_x, \\ \mathbf{a} &= a\mathbf{E}_x = \ddot{x}\mathbf{E}_x. \end{aligned} \tag{2}$$

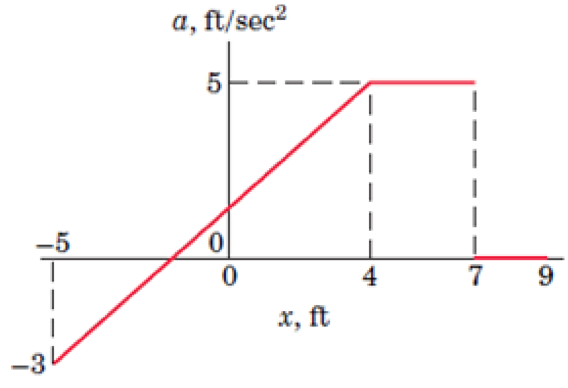
We can then relate the position, velocity, and acceleration of the particle in rectilinear motion through an expression without explicit dependence on time,

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}. \tag{3}$$

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 2/24]

2/24 A particle moving along a straight line has an acceleration which varies according to position as shown. If the velocity of the particle at the position $x = -5$ ft is $v = 4$ ft/sec, determine the velocity when $x = 9$ ft.



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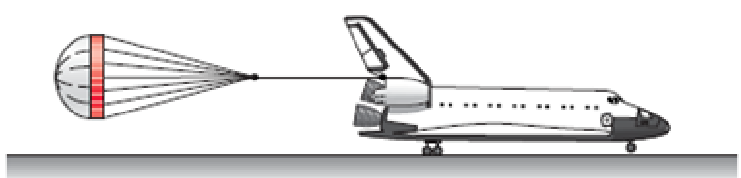
2. [MKB 2/28] Take the unit vector $\mathbf{E}_x$ to point along the horizontal. You may take the origin to coincide with the location of the plane when the parachute deploys (ie. when $v = 200$ mi/hr). Be careful to convert mi to ft and hr to sec.

2/28 The 230,000-lb space-shuttle orbiter touches down at about 220 mi/hr. At 200 mi/hr its drag parachute deploys. At 35 mi/hr, the chute is jettisoned from the orbiter. If the deceleration in feet per second squared during the time that the chute is deployed is $-0.0003v^2$ (speed v in feet per second), determine the corresponding distance traveled by the orbiter. Assume no braking from its wheel brakes.

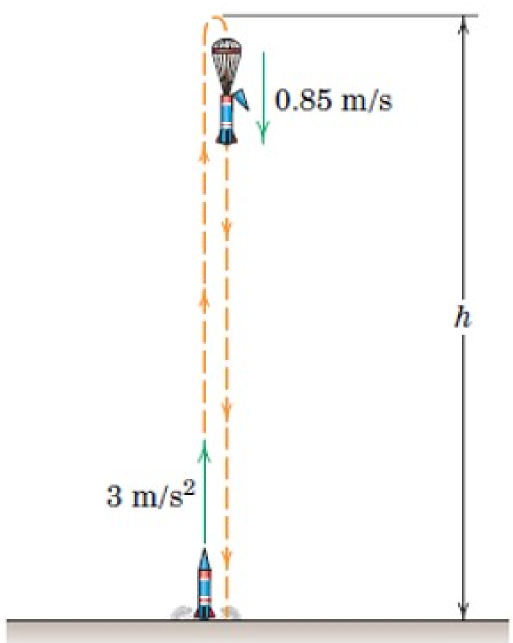

PROBLEM 2/28

3. [MKB 2/25] Take the unit vector $\mathbf{E}_y$ to be pointed vertically upwards and take the origin to be located at the initial position of the rocket. In this case, the position vector of the rocket would be $\mathbf{r} = r\mathbf{E}_y$. You are given the acceleration in m s^{-2} to be

$$a = \begin{cases} 3 & 0 \leq t < 8 \\ -9.81 & 8 \leq t < t_{top} \\ 0 & t_{top} < t \leq t_{end} \end{cases}$$

where t_{top} is the time at which the rocket reaches the apex of the trajectory and t_{end} is the time at which the rocket comes backs to its initial location.

2/25 A model rocket is launched from rest with a constant upward acceleration of 3 m/s^2 under the action of a small thruster. The thruster shuts off after 8 seconds, and the rocket continues upward until it reaches its apex. At apex, a small chute opens which ensures that the rocket falls at a constant speed of 0.85 m/s until it impacts the ground. Determine the maximum height h attained by the rocket and the total flight time. Neglect aerodynamic drag during ascent, and assume that the mass of the rocket and the acceleration of gravity are both constant.



PROBLEM 2/25

Note that this problem seems to assume that the velocity at the apex changes very quickly from 0 m/s to 0.85 m/s downwards, so we will neglect this transition.