MECH230 - Fall 2024
Recommended Problems - Set 03

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Kinematics in Cylindrical Polar Coordinates Define the polar basis $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{E}_z\}$ where

$$\mathbf{e}_r = \cos(\theta)\mathbf{E}_x + \sin(\theta)\mathbf{E}_y, \quad \text{and} \quad \mathbf{e}_\theta = -\sin(\theta)\mathbf{E}_x + \cos(\theta)\mathbf{E}_y. \tag{1}$$

Notice that

$$\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta, \quad \text{and} \quad \dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r. \tag{2}$$

The position, velocity, and acceleration vectors are expressed in this basis as

$$\begin{aligned}
\mathbf{r} &= r\mathbf{e}_r + z\mathbf{E}_z, \\
\mathbf{v} &= \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{E}_z, \\
\mathbf{a} &= \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta + \ddot{z}\mathbf{E}_z.
\end{aligned} \tag{3}$$

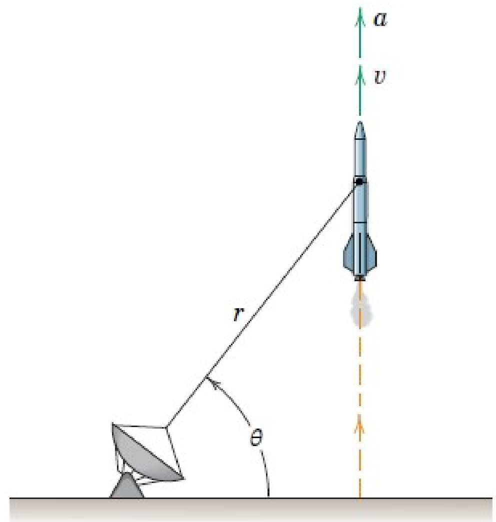
Kinetics in Polar Coordinates Projecting the balance of linear momentum, $\mathbf{F} = m\mathbf{a}$ on the polar basis vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{E}_z$ yields respectively to

$$\begin{aligned}
\mathbf{F} \cdot \mathbf{e}_r &= m\left(\ddot{r} - r\dot{\theta}^2\right), \\
\mathbf{F} \cdot \mathbf{e}_\theta &= m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right), \\
\mathbf{F} \cdot \mathbf{E}_z &= m\ddot{z}.
\end{aligned} \tag{4}$$

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 2/021] Take the origin to be at the satellite and $\mathbf{E}_x$ and $\mathbf{E}_y$ to point rightwards and upwards respectively. Write the position of the rocket both in cylindrical and Cartesian coordinates.

2/121 **SS** **The rocket is fired vertically and tracked by the radar station shown. When θ reaches 60°, other corresponding measurements give the values $r = 9$ km, $\ddot{r} = 21$ m/s^2, and $\dot{\theta} = 0.02$ rad/s. Calculate the magnitudes of the velocity and acceleration of the rocket at this position.**



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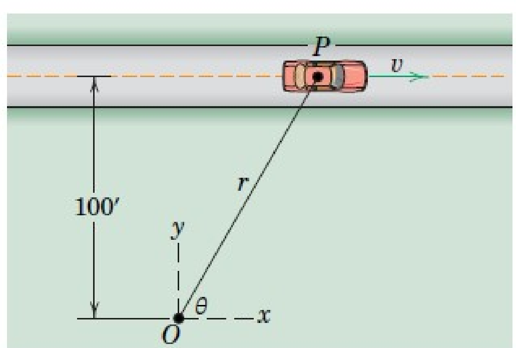
2. [MKB 02-105] Take $\mathbf{E}_x$ and $\mathbf{E}_y$ to point rightwards and upwards respectively. Write the position of the car both in cylindrical and Cartesian coordinates.

2/105 **A car P travels along a straight road with a constant speed $v = 65$ mi/hr. At the instant when the angle $\theta = 60°$, determine the values of $\dot{r}$ in ft/sec and $\dot{\theta}$ in deg/sec.**



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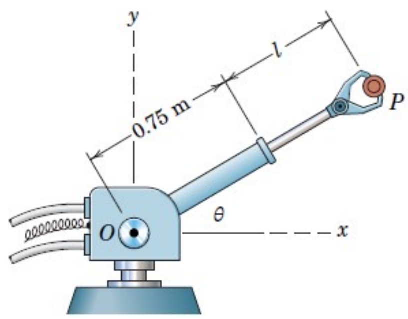
3. [MKB 02-126] Take $\mathbf{E}_x$ and $\mathbf{E}_y$ to point rightwards and upwards respectively. Write the position vector of P as $\mathbf{r} = (0.75 + \ell)\mathbf{e}_r$ m. Differentiate the position vector to obtain the velocity and acceleration vectors.

If this setup is in the vertical plane (i.e. $\mathbf{g} = -g\mathbf{E}_y$), what would be the force applied by the robot arm on part P having mass m at the instant described. Use the four steps in your analysis.

2/126 The robot arm is elevating and extending simultaneously. At a given instant, $\theta = 30°$, $\dot{\theta} = 10$ deg/s = constant, $l = 0.5$ m, $\dot{l} = 0.2$ m/s, and $\ddot{l} = -0.3$ m/s^2. Compute the magnitudes of the velocity **v** and acceleration **a** of the gripped part P. In addition, express **v** and **a** in terms of the unit vectors **i** and **j**.



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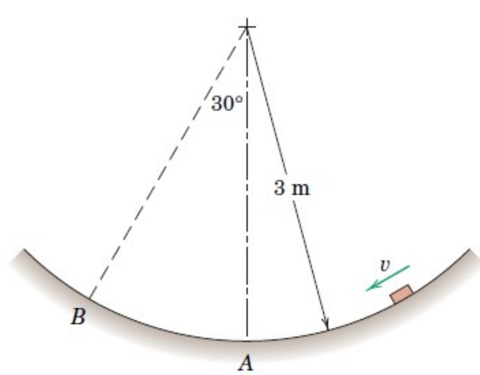
4. [MKB 03-037] In this problem, you need to use the 4 steps at points A and B independently. Take your origin to be at the center of curvature of the path (ie. at the center of the circle indicated by +). Take $\mathbf{E}_x$ to point vertically downward and $\mathbf{E}_y$ to point horizontally to the left. How would your calculations be affected if you took $\mathbf{E}_x$ and $\mathbf{E}_y$ to instead respectively point rightwards and upwards?

3/37 The small 0.6-kg block slides with a small amount of friction on the circular path of radius 3 m in the vertical plane. If the speed of the block is 5 m/s as it passes point A and 4 m/s as it passes point B, determine the normal force exerted on the block by the surface at each of these two locations.



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