MECH230 - Fall 2024
Recommended Problems - Set 05

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Kinematics in the Serret-Frenet Basis Define the Serret-Frenet basis $\{\mathbf{e}_t, \mathbf{e}_n, \mathbf{e}_b\}$ as

$$v = ||\mathbf{v}|| = \frac{ds}{dt}, \quad \mathbf{e}_t = \frac{\mathbf{v}}{v}, \quad \frac{d\mathbf{e}_t}{ds} = \kappa \mathbf{e}_n, \quad \mathbf{e}_b = \mathbf{e}_t \times \mathbf{e}_n, \quad \frac{d\mathbf{e}_b}{ds} = -\tau \mathbf{e}_n, \tag{1}$$

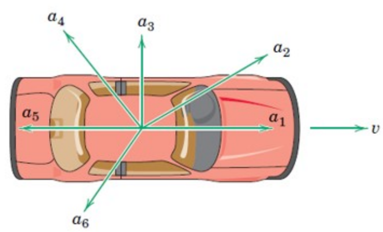
where κ is the paths' curvature, $\rho = \frac{1}{\kappa}$ is the path's radius of curvature, and τ is the path's torsion. The velocity and acceleration vectors in this basis are expressed as

$$\begin{aligned} \mathbf{v} &= v\mathbf{e}_t, \\ \mathbf{a} &= \dot{v}\mathbf{e}_t + \kappa v^2 \mathbf{e}_n. \end{aligned} \tag{2}$$

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 2/079] Conduct the following analysis while referring to the representation of the acceleration vector on the Serret-Frenet basis.

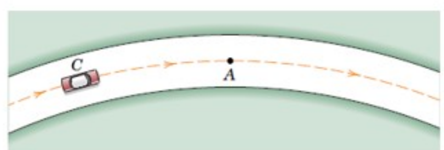
2/79 Six acceleration vectors are shown for the car whose velocity vector is directed forward. For each acceleration vector describe in words the instantaneous motion of the car.



PROBLEM 2/79

2. [MKB 02-090]

2/90 The car C increases its speed at the constant rate of 1.5 m/s^2 as it rounds the curve shown. If the magnitude of the total acceleration of the car is 2.5 m/s^2 at point A where the radius of curvature is 200 m, compute the speed v of the car at this point.



PROBLEM 2/90

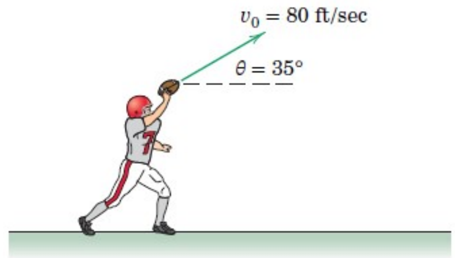
3. [MKB 02-091]

2/91 At the bottom A of the vertical inside loop, the magnitude of the total acceleration of the airplane is $3g$. If the airspeed is 800 km/h and is increasing at the rate of 20 km/h per second, calculate the radius of curvature ρ of the path at A.

PROBLEM 2/91

4. [02-097]

2/97 A football player releases a ball with the initial conditions shown in the figure. Determine the radius of curvature ρ of the path and the time rate of change $\dot{v}$ of the speed at times $t = 1$ sec and $t = 2$ sec, where $t = 0$ is the time of release from the quarterback's hand.



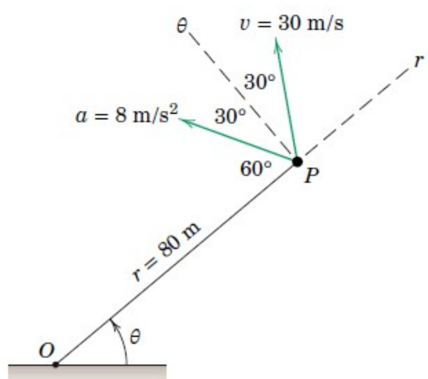
PROBLEM 2/97

5. [02-103]

▶**2/103** A particle which moves with curvilinear motion has coordinates in meters which vary with time t in seconds according to $x = 2t^2 + 3t - 1$ and $y = 5t - 2$. Determine the coordinates of the center of curvature C at time $t = 1$ s.

6. [02-199]

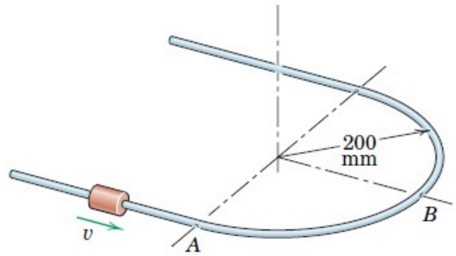
2/199 At the instant depicted, assume that the particle P, which moves on a curved path, is 80 m from the pole O and has the velocity v and acceleration a as indicated. Determine the instantaneous values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, $\ddot{\theta}$, the n- and t-components of acceleration, and the radius of curvature ρ.



PROBLEM 2/199

7. [03-040]

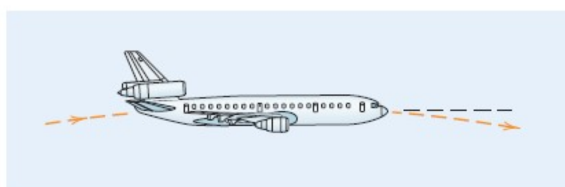
3/40 The 120-g slider has a speed $v = 1.4$ m/s as it passes point A of the smooth guide, which lies in a horizontal plane. Determine the magnitude R of the force which the guide exerts on the slider (a) just before it passes point A of the guide and (b) as it passes point B.



PROBLEM 3/40

8. [03-041]

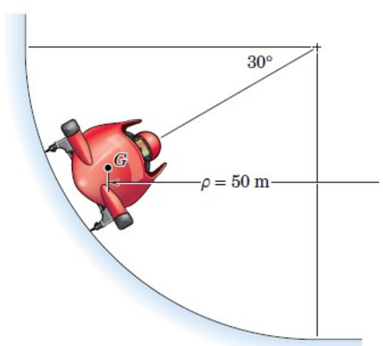
3/41 A jet transport plane flies in the trajectory shown in order to allow astronauts to experience the "weightless" condition similar to that aboard orbiting spacecraft. If the speed at the highest point is 600 mi/hr, what is the radius of curvature ρ necessary to exactly simulate the orbital "free-fall" environment?



PROBLEM 3/41

9. [03-043] Be careful with drawing the Serret-Frenet basis in this problem.

3/43 Determine the speed which the 630-kg four-man bobsled must have in order to negotiate the turn without reliance on friction. Also find the net normal force exerted on the bobsled by the track.



PROBLEM 3/43