MECH230 - Fall 2024
Recommended Problems - Set 06

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<u>Relative motions of translating rigid bodies.</u> The position of a body B with respect to a body A is written as $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A$. Taking derivatives of this equation, we get $\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$ and $\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$.

<u>Corotational vector</u> A corotational vector is a vector fixed to a rigid body and rotates with it.
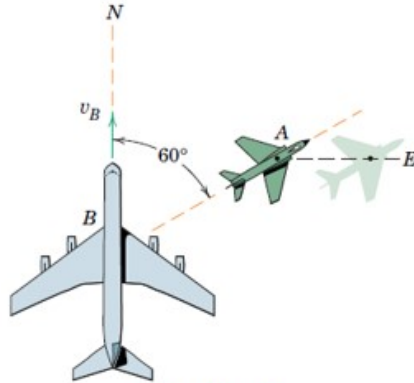
<u>The spring force</u> A spring of stiffness K with unstretched length ℓ_0 whose base is at point A and whose free end is attached to a mass m with position vector $\mathbf{r}$ applies a force on m that is

$$\mathbf{F}_s = -K(||\mathbf{r} - \mathbf{r}_A|| - \ell_0)\frac{\mathbf{r} - \mathbf{r}_A}{||\mathbf{r} - \mathbf{r}_A||}. \tag{1}$$

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 2/051]

PROBLEM 2/151

2. [MKB 02-141] Draw the position vector from O to P and cylindrical polar coordinate system at point P. As the base of the robot rotates, $\mathbf{E}_z$ remains fixes, but the basis vectors $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ rotate with $\dot\theta = \omega = const$. Define a unit vector $\mathbf{e}_1$ pointing from O to A and a unit vector $\mathbf{e}_2$ perpendicular to it pointing from P to A. Express the $\{\mathbf{e}_1, \mathbf{e}_2\}$ vectors in terms of $\{\mathbf{e}_r, \mathbf{E}_z\}$ and the angle β. You can then write the position vector of P as $\mathbf{r}_P = 0.3\mathbf{e}_1 - 0.2\mathbf{e}_2$ and differentiate this expression to get the velocity and acceleration of P.

2/141 An industrial robot is being used to position a small part P. Calculate the magnitude of the acceleration $\mathbf{a}$ of P for the instant when $\beta = 30°$ if $\dot\beta = 10$ deg/s and $\ddot\beta = 20$ deg/s² at this same instant. The base of the robot is revolving at the constant rate $\omega = 40$ deg/s. During the motion arms AO and AP remain perpendicular.
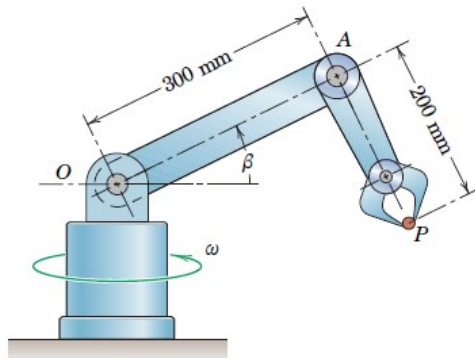
PROBLEM 2/141

3. [MKB 02-166] Draw your Cartesian and polar bases carefully! Remember that for the setup we introduced in class, the angle θ is the angle between $\mathbf{E}_x$ and $\mathbf{e}_r$.

2/166 Car A is traveling at the constant speed of 60 km/h as it rounds the circular curve of 300-m radius and at the instant represented is at the position $\theta = 45°$. Car B is traveling at the constant speed of 80 km/h and passes the center of the circle at this same instant. Car A is located with respect to car B by polar coordinates r and θ with the pole moving with B. For this instant determine $v_{A/B}$ and the values of $\dot{r}$ and $\dot{\theta}$ as measured by an observer in car B.
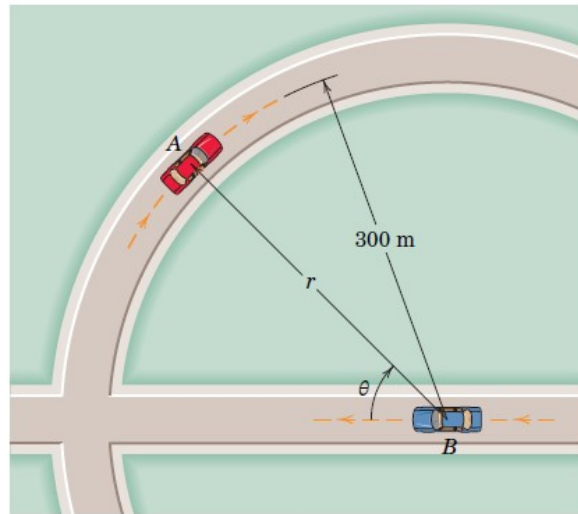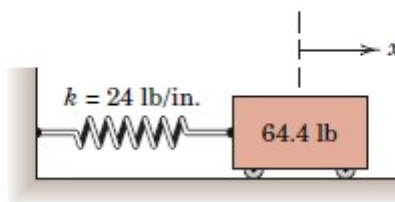

PROBLEM 2/166

In the following problems from chapter 8, find the equation of motion of the block in each case. Neglect what the original MKB question is asking.

4. [08-002]

8/2 Determine the natural frequency of the spring-mass system in both rad/sec and cycles/sec (Hz).
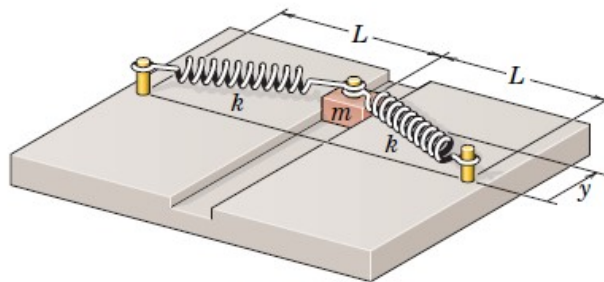

PROBLEM 8/2

5. [08-004]

8/4 For the spring-mass system shown, determine the static deflection δ_{st}, the system period τ, and the maximum velocity v_{max} which result if the cylinder is displaced 100 mm downward from its equilibrium position and released from rest.

$k = 98$ N/m

Equilibrium position

$m = 2$ kg

y
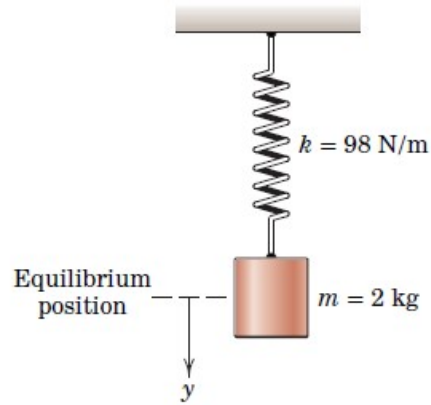
PROBLEM 8/4

6. [08-019]

▶**8/19** The slider of mass m is confined to the horizontal slot shown. The two springs each of constant k are linear. Derive the nonlinear equation of motion for small values of y, retaining terms of order y^3 and larger. Both springs are unstretched when $y = 0$. Neglect friction.

L

L

k

m

k

y

PROBLEM 8/19