MECH230 - Fall 2024
Recommended Problems - Set 09

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Linear impulse - linear momentum equation Integrating the balance of linear momentum with respect to time yields the linear impulse - linear momentum equation

$$\int_{t_A}^{t_B} \mathbf{F} dt = \mathbf{G}_B - \mathbf{G}_A, \tag{1}$$

where $\int_{t_A}^{t_B} \mathbf{F} dt$ is termed the linear impulse of a force $\mathbf{F}$. Here, $\mathbf{G}_A = \mathbf{G}(t_A)$ and $\mathbf{G}_B = \mathbf{G}(t_B)$.

Conservation of linear momentum If $\mathbf{F} = 0$, then the linear momentum of the system is conserved $\mathbf{G}_A = \mathbf{G}_B$. The linear momentum is conserved in a certain unit direction $\mathbf{c}$ only if $\mathbf{F} \cdot \mathbf{c} + \mathbf{G} \cdot \dot{\mathbf{c}} = \mathbf{0}$. If $\mathbf{c}$ is a constant, this expression simplifies to $\mathbf{F} \cdot \mathbf{c} = \mathbf{0}$.

Angular momentum of a particle The angular momentum of a particle about the fixed origin O is defined to be

$$\mathbf{H}^O = \mathbf{r} \times m\mathbf{v}. \tag{2}$$

Moment The moment of a force $\mathbf{F}$ applied at point A about point P is the position vector from P to a point on the line of action of $\mathbf{F}$ crossed with $\mathbf{F}$

$$\mathbf{M}^P = (\mathbf{r}_A - \mathbf{r}_P) \times \mathbf{F}. \tag{3}$$

Balance of angular momentum of a particle Crossing $\mathbf{r}$ with the balance of linear momentum equation, we obtain the balance of angular momentum equation for a particle

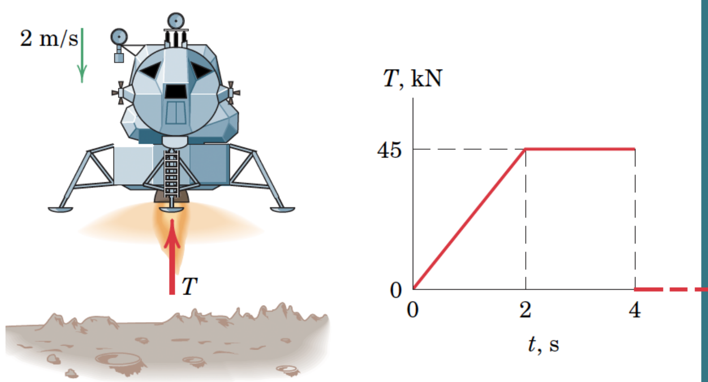
$$\mathbf{M}^O = \dot{\mathbf{H}}^O. \tag{4}$$

Conservation of angular momentum If $\mathbf{M}^O = \mathbf{O}$, then the angular momentum of the system about point O, $\mathbf{H}^O$, is conserved. The linear momentum is conserved in a certain unit direction $\mathbf{c}$ only if $\mathbf{M}^O \cdot \mathbf{c} + \mathbf{H}^O \cdot \dot{\mathbf{c}} = \mathbf{0}$. If $\mathbf{c}$ is a constant, this expression simplifies to $\mathbf{M}^O \cdot \mathbf{c} = \mathbf{0}$.

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 03-149] This problem is a straightforward application of the Linear Impulse - Linear Momentum equation.

3/149 The 15 200-kg lunar lander is descending onto the moon's surface with a velocity of 2 m/s when its retro-engine is fired. If the engine produces a thrust T for 4 s which varies with time as shown and then cuts off, calculate the velocity of the lander when $t = 5$ s, assuming that it has not yet landed. Gravitational acceleration at the moon's surface is 1.62 m/s^2.



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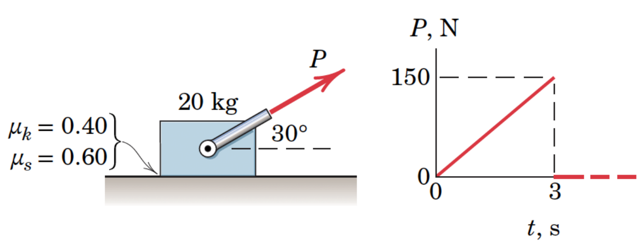
2. [MKB 03-159] All you need to know from the previous problem is that the block is is subjected to the time-varying horizontal force whose magnitude P is shown in the plot. Note that the force is zero for all times greater than 3 s.

3/159 All elements of the previous problem remain unchanged, except that the force P is now held at a constant 30° angle relative to the horizontal. Determine the time t_s at which the initially stationary 20-kg block comes to rest.



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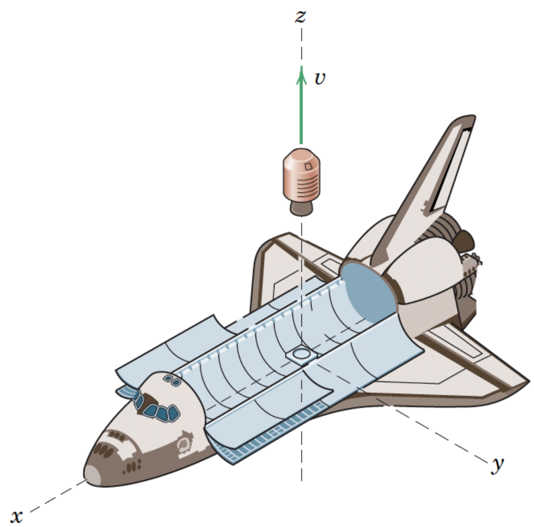
3. [MKB 03-161] You are going to use Newton's third law in this problem. Remember from math that $\int_{x_A}^{x_B} f(x)dx = F_{average}(x_B - x_A)$.

3/161 The space shuttle launches an 800-kg satellite by ejecting it from the cargo bay as shown. The ejection mechanism is activated and is in contact with the satellite for 4 s to give it a velocity of 0.3 m/s in the z-direction relative to the shuttle. The mass of the shuttle is 90 Mg. Determine the component of velocity v_f of the shuttle in the minus z-direction resulting from the ejection. Also find the time average F_{av} of the ejection force.



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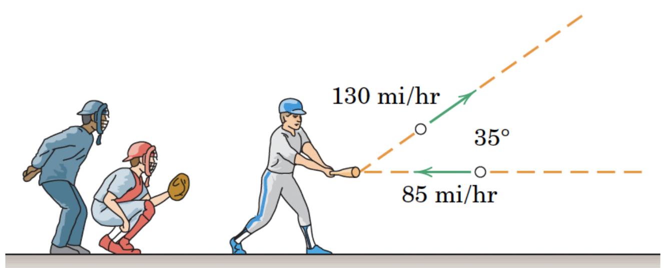
4. [03-167]

3/167 The baseball is traveling with a horizontal velocity of 85 mi/hr just before impact with the bat. Just after the impact, the velocity of the $5\frac{1}{8}$-oz ball is 130 mi/hr directed at 35° to the horizontal as shown. Determine the x- and y-components of the average force **R** exerted by the bat on the baseball during the 0.005-sec impact. Comment on the treatment of the weight of the baseball (a) during the impact and (b) over the first few seconds after impact.



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5. [03-177] This is a quintessential central force problem. What quantities of this system are conserved? Refer to table D2 at the end of this document for the radius of the earth.

3/177 Just after launch from the earth, the space-shuttle orbiter is in the 37 × 137–mi orbit shown. At the apogee point A, its speed is 17,290 mi/hr. If nothing were done to modify the orbit, what would be its speed at the perigee P? Neglect aerodynamic drag. (Note that the normal practice is to add speed at A, which raises the perigee altitude to a value that is well above the bulk of the atmosphere.)

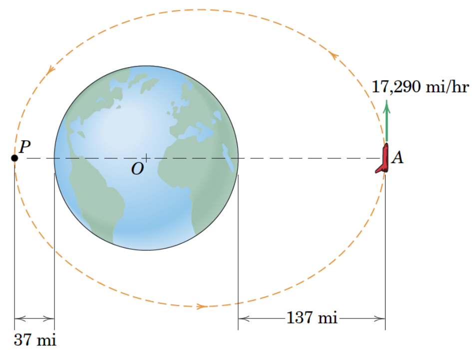

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TABLE D/2 Solar System Constants

Universal gravitational constant $G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$
$= 3.439(10^{-8})\ \text{ft}^4/(\text{lb-sec}^4)$

Mass of Earth $m_e = 5.976(10^{24})$ kg
$= 4.095(10^{23})\ \text{lb-sec}^2/\text{ft}$

Period of Earth's rotation (1 sidereal day) = 23 h 56 min 4 s
= 23.9344 h

Angular velocity of Earth $\omega = 0.7292(10^{-4})$ rad/s
Mean angular velocity of Earth–Sun line $\omega' = 0.1991(10^{-6})$ rad/s
Mean velocity of Earth's center about Sun = 107 200 km/h
= 66,610 mi/hr

Body	Mean Distance to Sun km (mi)	Eccentricity of Orbit e	Period of Orbit solar days	Mean Diameter km (mi)	Mass Relative to Earth	Surface Gravitational Acceleration m/s^2 (ft/sec^2)	Escape Velocity km/s (mi/sec)
Sun	—	—	—	1 392 000 (865 000)	333 000	274 (898)	616 (383)
Moon	384 398[1] (238 854)[1]	0.055	27.32	3 476 (2 160)	0.0123	1.62 (5.32)	2.37 (1.47)
Mercury	57.3×10^6 (35.6×10^6)	0.206	87.97	5 000 (3 100)	0.054	3.47 (11.4)	4.17 (2.59)
Venus	108×10^6 (67.2×10^6)	0.0068	224.70	12 400 (7 700)	0.815	8.44 (27.7)	10.24 (6.36)
Earth	149.6×10^6 (92.96×10^6)	0.0167	365.26	12 742[2] (7 918)[2]	1.000	9.821[3] (32.22)[3]	11.18 (6.95)
Mars	227.9×10^6 (141.6×10^6)	0.093	686.98	6 788 (4 218)	0.107	3.73 (12.3)	5.03 (3.13)
Jupiter[4]	778×10^6 (483×10^6)	0.0489	4333	139 822 (86 884)	317.8	24.79 (81.3)	59.5 (36.8)

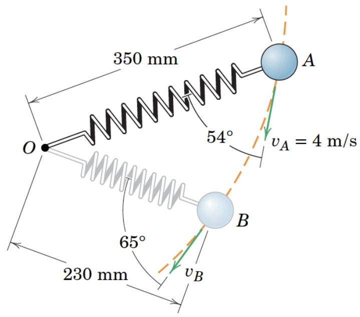
[1]Mean distance to Earth (center-to-center)
[2]Diameter of sphere of equal volume, based on a spheroidal Earth with a polar diameter of 12 714 km (7900 mi) and an equatorial diameter of 12 756 km (7926 mi)
[3]For nonrotating spherical Earth, equivalent to absolute value at sea level and latitude 37.5°
[4]Note that Jupiter is not a solid body.

6. [03-181]

3/181 A particle with a mass of 4 kg has a position vector in meters given by $\mathbf{r} = 3t^2\mathbf{i} - 2t\mathbf{j} - 3t\mathbf{k}$, where t is the time in seconds. For $t = 3$ s determine the magnitude of the angular momentum of the particle and the magnitude of the moment of all forces on the particle, both about the origin of coordinates.

7. [03-185]

3/185 A particle of mass m moves with negligible friction on a horizontal surface and is connected to a light spring fastened at O. At position A the particle has the velocity $v_A = 4$ m/s. Determine the velocity v_B of the particle as it passes position B.



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8. [03-192] To avoid confusion label r in the figure R and the angle θ requested in the solution as β.

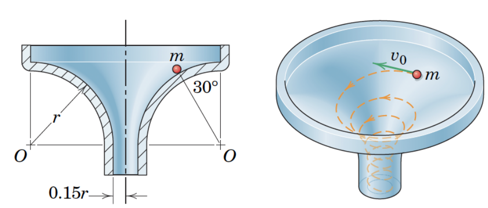
Step 1. Choose the origin O to be at the bottom of the funnel and setup the cylinderical-polar coordinate system. Derive $\mathbf{v}$ but not $\mathbf{a}$, we will not need it as we will solve the problem by exploiting conservations. The particle is constrained to move on a surface of revolution given by $z^2+(r-1.15R)^2 = R^2$. A time derivative of this expression yields $z\dot{z} + \dot{r}\,(r - 1.15R) = 0$.

Step 2. Draw a free-body diagram of the particle. Express the normal force as $\mathbf{N} = N\mathbf{n}$, where $\mathbf{n}$ is a unit direction normal to the surface of revolution. In theory, $\mathbf{n}$ could be computed from a gradient of $z^2 + (r - 1.15R)^2 = R^2$, but you don't need to do that here. You only need to note that $\mathbf{N}$ has $\mathbf{e}_r$ and $\mathbf{E}_z$ components.

Step 3. In Step III, prove a conservation on the total mechanical energy E and a conservation of $\mathbf{E}_z$ components of the angular momentum $\mathbf{H}^O$. You will need to refer to your FBD to identify these conserved quantities.

Step 4. Calculate the numerical values of E and and $\mathbf{H}_O \cdot \mathbf{E}_x$ using the initial conditions and complete your analysis.

3/192 A particle is launched with a horizontal velocity $v_0 = 0.55$ m/s from the 30° position shown and then slides without friction along the funnel-like surface. Determine the angle θ which its velocity vector makes with the horizontal as the particle passes level O-O. The value of r is 0.9 m.



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