# MECH230 - Fall 2024 Recommended Problems - Set 12

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Angular velocity and angular acceleration Consider the reference and current configuration of a rigid body in general plane motion. At any time t, any material line on the rigid body in the current configuration makes the same angle  $\theta$  with its precursor in the reference configuration. Define the angular velocity of the rigid body to be  $\boldsymbol{\omega} = \dot{\theta} \mathbf{E}_z$  where  $\mathbf{E}_z$  is the fixed axes of rotation.  $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$  is the angular acceleration of the rigid body.

<u>Corotational basis</u> Let the orthonormal basis  $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z = \mathbf{E}_z\}$  be a corotational basis that is fixed on and corotates with the rigid body such that

$$\mathbf{e}_x = \cos(\theta)\mathbf{E}_x + \sin(\theta)\mathbf{E}_y,\tag{1}$$

$$\mathbf{e}_y = -\sin(\theta)\mathbf{E}_x + \cos(\theta)\mathbf{E}_y.$$
 (2)

Then,  $\dot{\mathbf{e}}_x = \boldsymbol{\omega} \times \mathbf{e}_x$  and  $\dot{\mathbf{e}}_y = \boldsymbol{\omega} \times \mathbf{e}_y$ .

Velocity and acceleration analysis for material points on a RB Consider two material points A and B with

$$\mathbf{r}_B - \mathbf{r}_A = x\mathbf{e}_x + y\mathbf{e}_y. \tag{3}$$

As the rigid body moves, x and y remain constants, hence

$$\mathbf{v}_B - \mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_B - \mathbf{r}_A,\tag{4}$$

$$\mathbf{a}_B - \mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_B - \mathbf{r}_A + \boldsymbol{\omega} \times \mathbf{v}_B - \mathbf{v}_A.$$
(5)

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

## 1. [MKB 05-010]

5/10 The device shown rotates about the fixed z-axis with angular velocity  $\omega = 20$  rad/s and angular acceleration  $\alpha = 40$  rad/s<sup>2</sup> in the directions indicated. Determine the instantaneous velocity and acceleration of point *B*.



## 2. [MKB 05-013]

5/13 The bent flat bar rotates about a fixed axis through point O with the instantaneous angular properties indicated in the figure. Determine the velocity and acceleration of point A.



## 3. [05-018]

**5/18** Point A of the circular disk is at the angular position  $\theta = 0$  at time t = 0. The disk has angular velocity  $\omega_0 = 0.1$  rad/s at t = 0 and subsequently experiences a constant angular acceleration  $\alpha = 2$  rad/s<sup>2</sup>. Determine the velocity and acceleration of point A in terms of fixed **i** and **j** unit vectors at time t = 1 s.



4. [05-049]

**5/49** The cart has a velocity of 4 ft/sec to the right. Determine the angular speed N of the wheel so that point A on the top of the rim has a velocity (a) equal to 4 ft/sec to the left, (b) equal to zero, and (c) equal to 8 ft/sec to the right.



#### 5. [05-065]

5/65 The elements of a simplified clam-shell bucket for a dredge are shown. The cable which opens and closes the bucket passes through the block at *O*. With *O* as a fixed point, determine the angular velocity  $\omega$  of the bucket jaws when  $\theta = 45^{\circ}$  as they are closing. The upward velocity of the control cable is 0.5 m/s as it passes through the block.



### 6. [05-069]

5/69 **SS** A four-bar linkage is shown in the figure (the ground "link" OC is considered the fourth bar). If the drive link OA has a counterclockwise angular velocity  $\omega_0 = 10$  rad/s, determine the angular velocities of links AB and BC.

