MECH230 - Fall 2024 Recommended Problems - Set 13

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Sine law and cosine law



Referring to the above triangle with interior angles A, B, and C and with side lengths a, b, and c, the sine law and cosine law are respectively

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c},\tag{1}$$

$$c^{2} = a^{2} + b^{2} - 2ab\cos(C).$$
(2)

Instantaneous center of rotation If a rigid body is rotating about a fixed point O, the velocity vector \mathbf{v}_P of any point P on the rigid body is perpendicular to the position vector from O to P

$$\mathbf{v}_P \perp \left(\mathbf{v}_P - \mathbf{v}_O \right). \tag{3}$$

A body in general plane motion at an instant to has an instantaneous center of rotation (IC) such that $\mathbf{v}_{IC} = 0$ but $\mathbf{a}_{IC} \neq 0$ because the center of rotation moves in space. If the velocities of two points on a rigid body are known, IC lies on the intersection point of the two perpendiculars to the velocity vectors at these points.

<u>A particle in a noninertial frame</u> Considered two particles A and B with A being fixed to the rigid body and B moving with respect to the rigid body, then

$$\mathbf{r}_B - \mathbf{r}_A = x\mathbf{e}_x + y\mathbf{e}_y,\tag{4}$$

$$\mathbf{v}_B - \mathbf{v}_A = \boldsymbol{\omega} \times (\mathbf{r}_B - \mathbf{r}_A) + \mathbf{v}_{rel},\tag{5}$$

$$\mathbf{a}_B - \mathbf{a}_A = \boldsymbol{\alpha} \times (\mathbf{r}_B - \mathbf{r}_A) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times (\mathbf{r}_B - \mathbf{r}_A)) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}, \tag{6}$$

where

$$\mathbf{v}_{rel} = \dot{x}\mathbf{e}_x + \dot{y}\mathbf{e}_y, \quad \text{and} \quad \mathbf{a}_{rel} = \ddot{x}\mathbf{e}_x + \ddot{y}\mathbf{e}_y.$$
 (7)

The Coriolis acceleration is $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$ and the centripetal acceleration is $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{v}_{rel})$.

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These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 05-102]

5/102 The bar of Prob. 5/66 is repeated here. The ends of the 0.4-m bar remain in contact with their respective support surfaces. End *B* has a velocity of 0.5 m/s and an acceleration of 0.3 m/s^2 in the directions shown. Determine the angular acceleration of the bar and the acceleration of end *A*.



5/125 The disk rotates with angular speed $\omega = 2$ rad/s. The small ball A is moving along the radial slot with speed u = 100 mm/s relative to the disk. Determine the absolute velocity of the ball and state the angle β between this velocity vector and the positive x-axis.



3. [05-143]

5/143 The disk rotates about a fixed axis through point O with a clockwise angular velocity $\omega_0 = 20$ rad/s and a counterclockwise angular acceleration $\alpha_0 = 5$ rad/s² at the instant under consideration. The value of r is 200 mm. Pin A is fixed to the disk but slides freely within the slotted member BC. Determine the velocity and acceleration of A relative to slotted member BC and the angular velocity and angular acceleration of BC.

