TABLE D/3 Properties of Plane Figures

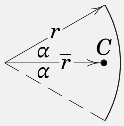
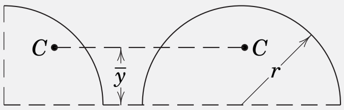
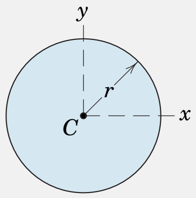
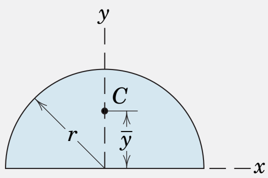
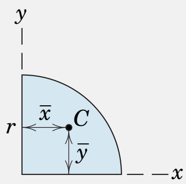
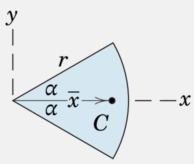
Figure	Centroid	Area Moments of Inertia
Arc Segment 	$\bar{r} = \dfrac{r \sin \alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \dfrac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \dfrac{\pi r^4}{4}$ $I_z = \dfrac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{8}$ $\bar{I}_x = \left(\dfrac{\pi}{8} - \dfrac{8}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \dfrac{2}{3} \dfrac{r \sin \alpha}{\alpha}$	$I_x = \dfrac{r^4}{4}\left(\alpha - \dfrac{1}{2} \sin 2\alpha\right)$ $I_y = \dfrac{r^4}{4}\left(\alpha + \dfrac{1}{2} \sin 2\alpha\right)$ $I_z = \dfrac{1}{2} r^4 \alpha$

TABLE D/3 Properties of Plane Figures ***Continued***

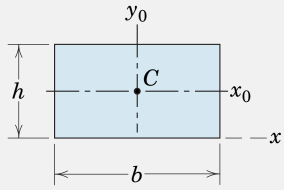
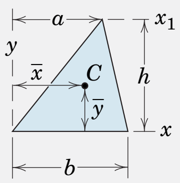
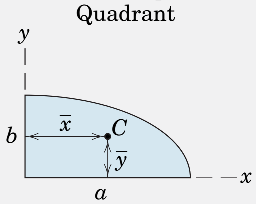
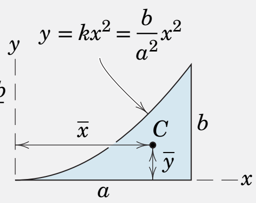
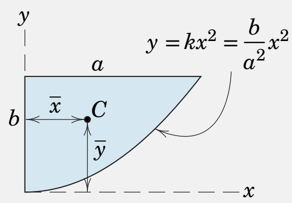
Figure	Centroid	Area Moments of Inertia
Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area 	$\bar{x} = \frac{a + b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}, \bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3 b}{16}, \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ $I_z = \frac{\pi ab}{16}(a^2 + b^2)$
Subparabolic Area Area $A = \frac{ab}{3}$ 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3 b}{5}$ $I_z = ab\left(\frac{a^2}{5} + \frac{b^2}{21}\right)$
Parabolic Area Area $A = \frac{2ab}{3}$ 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3 b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

TABLE D/4 Properties of Homogeneous Solids

(m = mass of body shown)

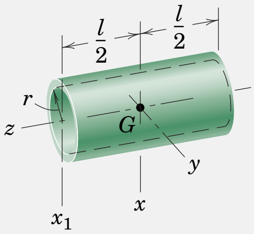
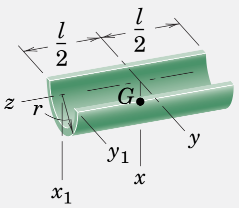
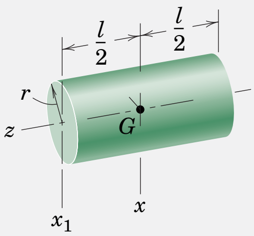
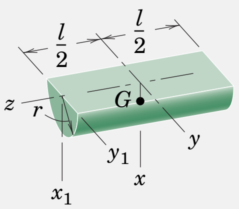
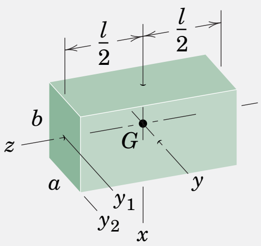
<table>
<tr><th>Body</th><th></th><th>Mass Center</th><th>Mass Moments of Inertia</th></tr>
<tr><td>
</td><td>Circular Cylindrical Shell</td><td>—</td><td>$I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$
$I_{x_1x_1} = \frac{1}{2}mr^2 + \frac{1}{3}ml^2$
$I_{zz} = mr^2$</td></tr>
<tr><td>
</td><td>Half Cylindrical Shell</td><td>$\bar{x} = \frac{2r}{\pi}$</td><td>$I_{xx} = I_{yy}$
$= \frac{1}{2}mr^2 + \frac{1}{12}ml^2$
$I_{x_1x_1} = I_{y_1y_1}$
$= \frac{1}{2}mr^2 + \frac{1}{3}ml^2$
$I_{zz} = mr^2$
$\bar{I}_{zz} = \left(1 - \frac{4}{\pi^2}\right)mr^2$</td></tr>
<tr><td>
</td><td>Circular Cylinder</td><td>—</td><td>$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$
$I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$
$I_{zz} = \frac{1}{2}mr^2$</td></tr>
<tr><td>
</td><td>Semicylinder</td><td>$\bar{x} = \frac{4r}{3\pi}$</td><td>$I_{xx} = I_{yy}$
$= \frac{1}{4}mr^2 + \frac{1}{12}ml^2$
$I_{x_1x_1} = I_{y_1y_1}$
$= \frac{1}{4}mr^2 + \frac{1}{3}ml^2$
$I_{zz} = \frac{1}{2}mr^2$
$\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right) mr^2$</td></tr>
<tr><td>
</td><td>Rectangular Parallelepiped</td><td>—</td><td>$I_{xx} = \frac{1}{12}m(a^2 + l^2)$
$I_{yy} = \frac{1}{12}m(b^2 + l^2)$
$I_{zz} = \frac{1}{12}m(a^2 + b^2)$
$I_{y_1y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$
$I_{y_2y_2} = \frac{1}{3}m(b^2 + l^2)$</td></tr>
</table>

TABLE D/4 **Properties of Homogeneous Solids** ***Continued***

(m = mass of body shown)

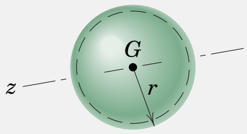
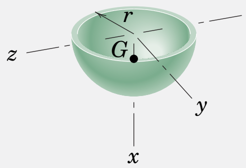
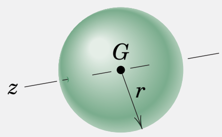
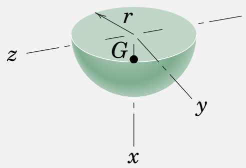
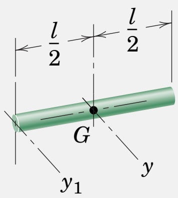
Body		Mass Center	Mass Moments of Inertia
	Spherical Shell	—	$I_{zz} = \frac{2}{3}mr^2$
	Hemispherical Shell	$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{5}{12}mr^2$
	Sphere	—	$I_{zz} = \frac{2}{5}mr^2$
	Hemisphere	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{83}{320}mr^2$
	Uniform Slender Rod	—	$I_{yy} = \frac{1}{12}ml^2$ $I_{y_1y_1} = \frac{1}{3}ml^2$

TABLE D/4 Properties of Homogeneous Solids *Continued*

(m = mass of body shown)

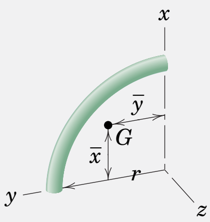
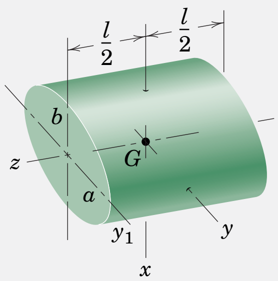
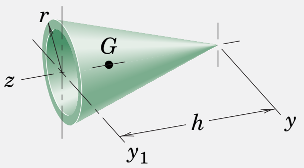
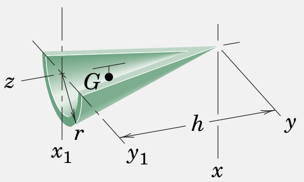
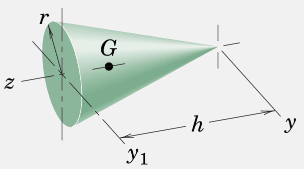
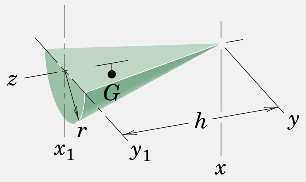
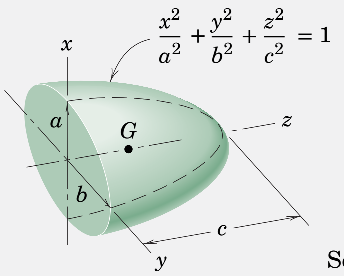
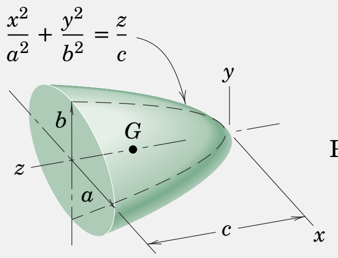
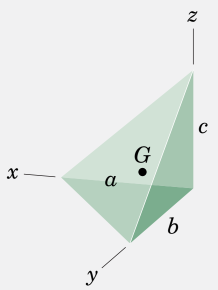
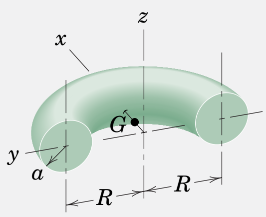
Body		Mass Center	Mass Moments of Inertia
	Quarter-Circular Rod	$\bar{x} = \bar{y} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2}mr^2$ $I_{zz} = mr^2$
	Elliptical Cylinder	—	$I_{xx} = \frac{1}{4}ma^2 + \frac{1}{12}ml^2$ $I_{yy} = \frac{1}{4}mb^2 + \frac{1}{12}ml^2$ $I_{zz} = \frac{1}{4}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{4}mb^2 + \frac{1}{3}ml^2$
	Conical Shell	$\bar{z} = \frac{2h}{3}$	$I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{yy} = \frac{1}{4}mr^2 + \frac{1}{18}mh^2$
	Half Conical Shell	$\bar{x} = \frac{4r}{3\pi}$ $\bar{z} = \frac{2h}{3}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
	Right Circular Cone	$\bar{z} = \frac{3h}{4}$	$I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{yy} = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$

TABLE D/4 Properties of Homogeneous Solids *Continued*

(m = mass of body shown)

Body	Mass Center	Mass Moments of Inertia
 Half Cone	$\bar{x} = \frac{r}{\pi}$ $\bar{z} = \frac{3h}{4}$	$I_{xx} = I_{yy}$ $= \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{zz} = \left(\frac{3}{10} - \frac{1}{\pi^2}\right)mr^2$
 Semiellipsoid	$\bar{z} = \frac{3c}{8}$	$I_{xx} = \frac{1}{5}m(b^2 + c^2)$ $I_{yy} = \frac{1}{5}m(a^2 + c^2)$ $I_{zz} = \frac{1}{5}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{5}m\left(b^2 + \frac{19}{64}c^2\right)$ $\bar{I}_{yy} = \frac{1}{5}m\left(a^2 + \frac{19}{64}c^2\right)$
 Elliptic Paraboloid	$\bar{z} = \frac{2c}{3}$	$I_{xx} = \frac{1}{6}mb^2 + \frac{1}{2}mc^2$ $I_{yy} = \frac{1}{6}ma^2 + \frac{1}{2}mc^2$ $I_{zz} = \frac{1}{6}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{6}m\left(b^2 + \frac{1}{3}c^2\right)$ $\bar{I}_{yy} = \frac{1}{6}m\left(a^2 + \frac{1}{3}c^2\right)$
 Rectangular Tetrahedron	$\bar{x} = \frac{a}{4}$ $\bar{y} = \frac{b}{4}$ $\bar{z} = \frac{c}{4}$	$I_{xx} = \frac{1}{10}m(b^2 + c^2)$ $I_{yy} = \frac{1}{10}m(a^2 + c^2)$ $I_{zz} = \frac{1}{10}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{3}{80}m(b^2 + c^2)$ $\bar{I}_{yy} = \frac{3}{80}m(a^2 + c^2)$ $\bar{I}_{zz} = \frac{3}{80}m(a^2 + b^2)$
 Half Torus	$\bar{x} = \frac{a^2 + 4R^2}{2\pi R}$	$I_{xx} = I_{yy} = \frac{1}{2}mR^2 + \frac{5}{8}ma^2$ $I_{zz} = mR^2 + \frac{3}{4}ma^2$