Pb1:

MECH420
Midterm exam / Fall 2023

a) The critical location is @ the fixed end A where the bending moment M_z is maximum.

$N_{max} = -P_{max} = -5KN$ (1)

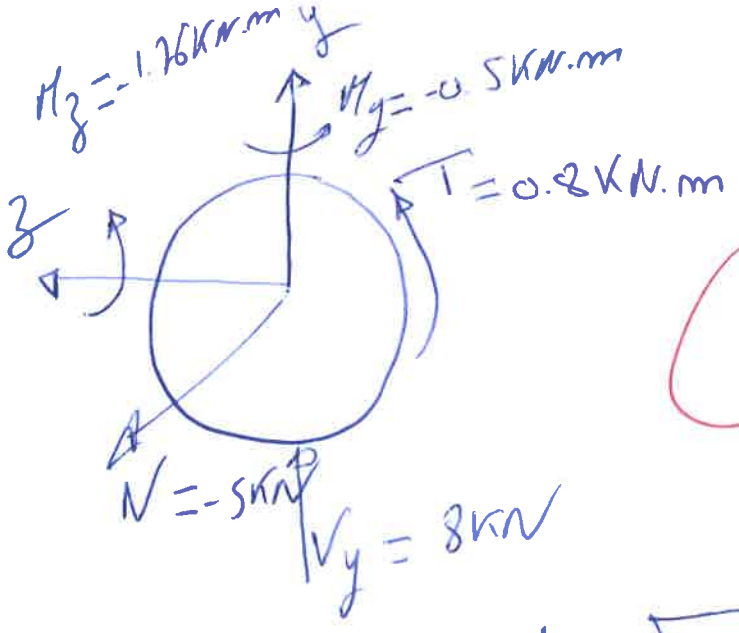
$(V_y)_{max} = F_{max} = 8KN$ (1)

$V_z = 0$ (1)

$T_{max} = -F_{max} \times 0.1$ (1)
$= -0.8\ KN.m$

$(M_y)_{max} = -P_{max} \times 0.1$ (1)
$= -0.5\ KN.m$

$(M_z)_{max} = -F_{max} \times 0.22$ (1)
$= -8 \times 0.22$
$= -1.76\ KN.m$



(2)

(2)

Resultant moment $M = \sqrt{M_y^2 + M_z^2} = 1.933KN.m$

maximum normal stress is located in the compression region because $N < 0$

$\sigma_{max} = \frac{N}{A} + \frac{Mc}{I} = \frac{-4 \times 5000}{\pi d^2} - \frac{1933 \times d/2}{\frac{\pi d^4}{64}}$ (2) (2)

$\Rightarrow \sigma_{max} = \frac{-6366.2}{d^2} - \frac{19689.38}{d^4}$ (1)

$$\tau_{max} = \frac{T \times c}{J} \quad (2) \qquad \frac{16T}{\pi d^3} = \frac{16 \times 800}{\pi d^3} = \frac{4074.37}{d^3}$$

Using maximum distortion energy theory

$$\sigma' = \sqrt{\sigma^2 + 3\tau^2} \quad (2)$$

$$\Rightarrow \sigma'_{max} = \sqrt{\left(\frac{6366.2}{d^2} + \frac{19689.37}{d^4}\right)^2 + 3 \times \left(\frac{4074.37}{d^3}\right)^2} \quad (3)$$

$$n = \frac{S_y}{\sigma'_{max}} = 3 \qquad (Pa)$$

safety factor

$$\Rightarrow \sigma'_{max} = \frac{S_y}{3} = \frac{300 \times 10^6}{3} \quad (2)$$

$$\Rightarrow \text{solving} \quad d \simeq 59\ mm$$

b) For $d = 60mm$

$$\begin{cases} \sigma_{max} = 88 MPa \\ \sigma_{min} = 0 \\ \tau_{max} = 18.86 MPa \\ \tau_{min} = 0 \end{cases}$$

$$\begin{cases} \sigma_a = K_f \left(\frac{\sigma_{max} - \sigma_{min}}{2}\right) \\ \sigma_m = K_f \left(\frac{\sigma_{max} + \sigma_{min}}{2}\right) \end{cases}$$

$$\begin{cases} \tau_a = K_{fs} \left(\frac{\tau_{max} - \tau_{min}}{2}\right) \\ \tau_m = K_{fs} \left(\frac{\tau_{max} + \tau_{min}}{2}\right) \end{cases}$$

$$\Rightarrow \sigma_a = 2 \times \frac{88}{2} = 88 MPa = \sigma_m$$

$$\tau_a = 1.5 \times \frac{18.86}{2} = 14.145 MPa = \tau_m \quad (2)$$

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = 91.39 \text{ MPa}$$

$$\sigma'_m = \sqrt{\sigma_m^2 + 3\tau_m^2} = 91.39 \text{ MPa}$$

Goodman: $$n = \left(\frac{\sigma'_a}{S_f} + \frac{\sigma'_m}{S_{ut}} \right)^{-1}$$

$$S_f = a N^b \quad a = \frac{(f \times S_{ut})^2}{S_e} \; ; \; b = -\frac{1}{3} \log\left(\frac{f S_{ut}}{S_e} \right)$$

$$f = 1.06 - 4.1 \times 10^{-4} \times 400 + 1.5 \times 10^{-5} \times (400)^2 = 0.92$$

$$S_e = K_a \times K_b \times K_c \times K_d \times K_e \times S'_e$$

$$S'_e = \frac{1}{2} S_{ut} = 200 \text{ MPa}$$

$$K_a = a S_{ut}^b \; ; \text{ machined} \Rightarrow \begin{cases} a = 3.04 \\ b = -0.217 \end{cases}$$

$$\Rightarrow K_a = 0.828$$

$$K_b = 1.51 \times 60^{-0.157} = 0.794$$

$$K_c = K_d = K_e = 1 \Rightarrow S_e = 131.49 \text{ MPa}$$

$$\Rightarrow a = 1030 \text{ MPa} \text{ and } b = -0.149$$

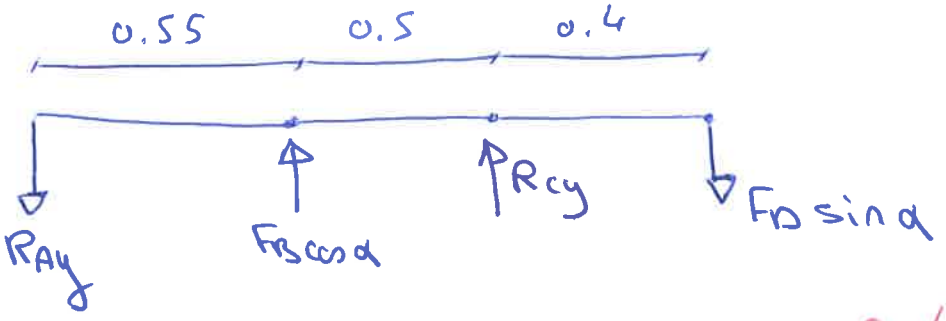
$$\Rightarrow S_f = 185.3 \text{ MPa}$$

$$\Rightarrow n = \left(\frac{91.39}{185.3} + \frac{91.39}{400} \right)^{-1} = 1.38$$

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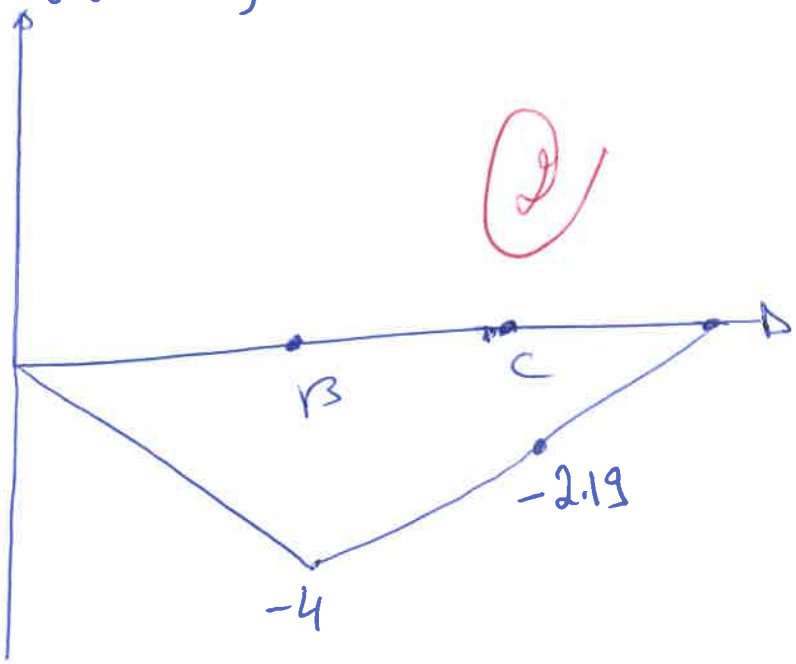
Pb 2:

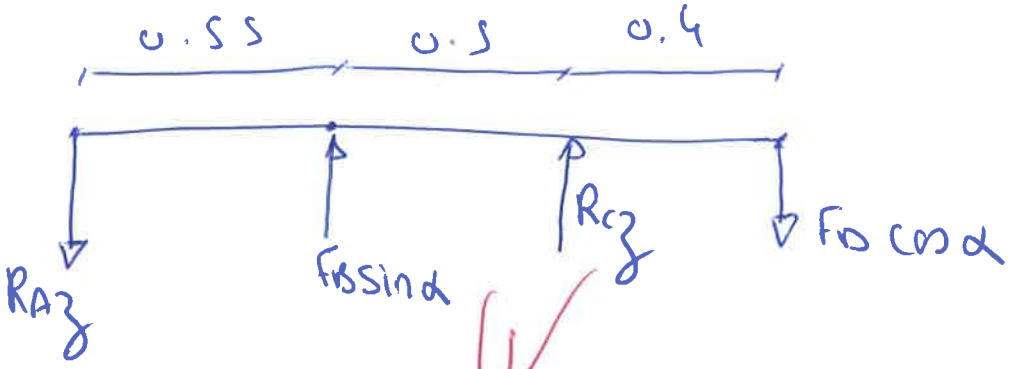


$R_{Ay} = (F_B \sin\alpha \times 0.4) + (F_B \cos\alpha \times 0.45)$ ✓ $= 7.263\,KN \downarrow$

$R_{cy} = 1.46\,KN \uparrow$ ✓

$M_z\,(KN.m)$

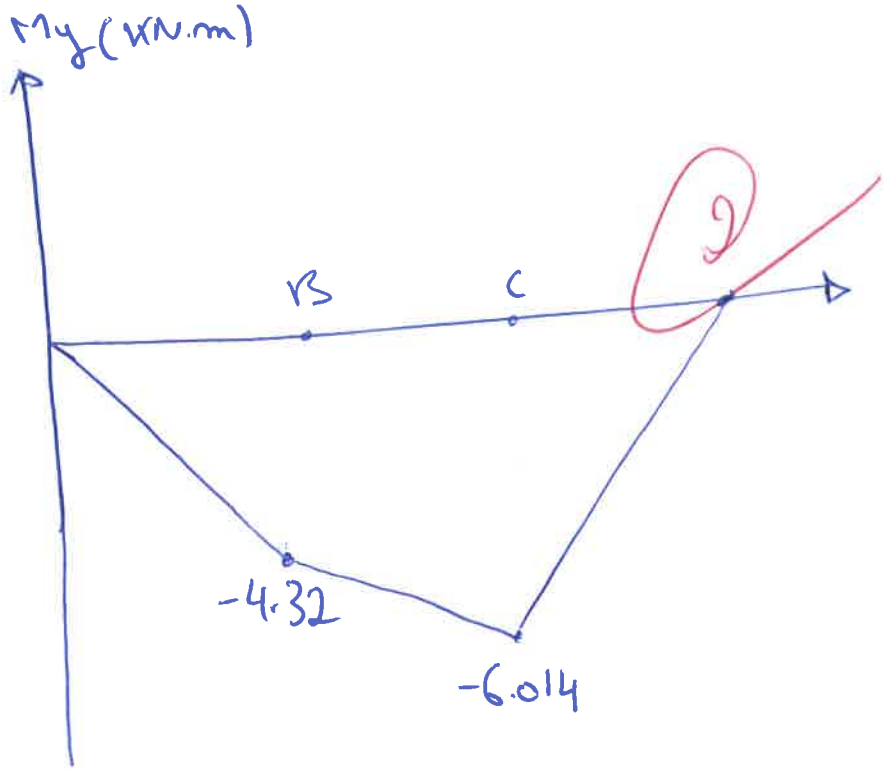




$R_{Az} = 7.86\,KN$ ✓

$R_{cz} = 18.8\,KN$ ✓

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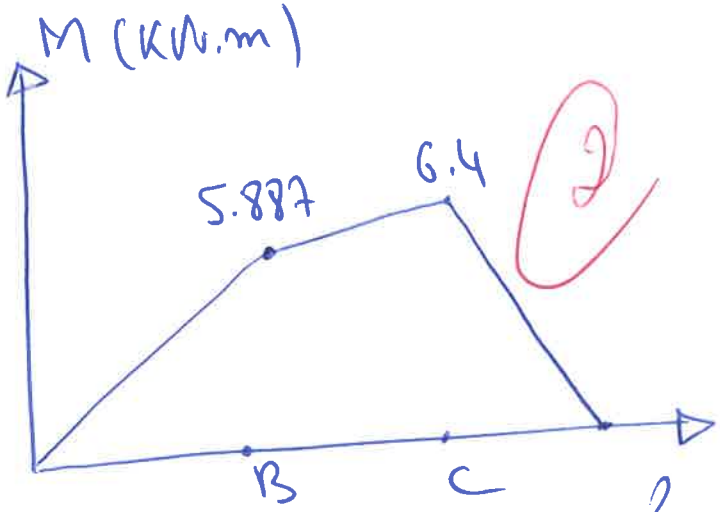
Resultant bending moment diagram:

$M_B = \sqrt{4^2 + 4.32^2} = 5.887\ KN.m$

$M_C = \sqrt{2.19^2 + 6.014^2} = 6.4\ KN.m$

1

1



2

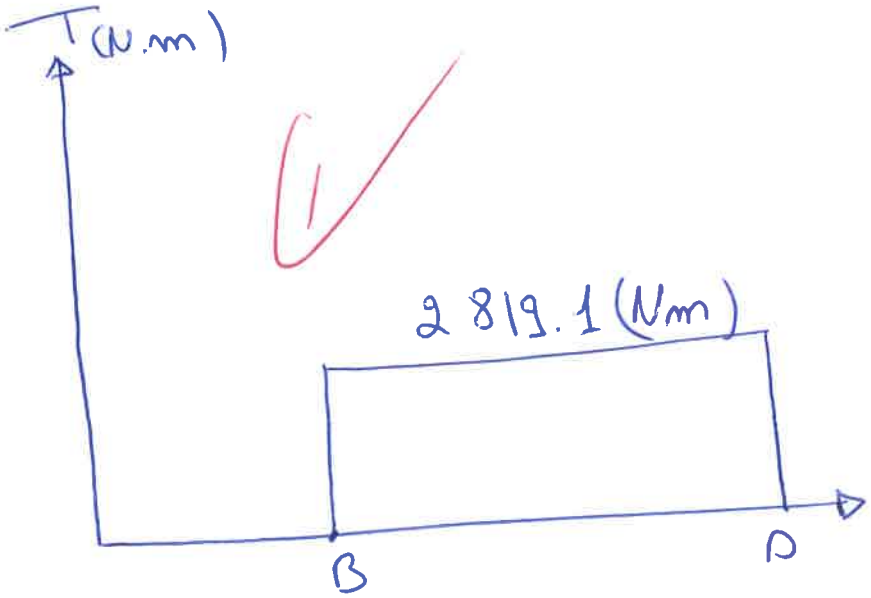
⇒ Critical locations should be considered:

- Key @ B
- shoulder fillet @ C (more critical than the shoulder @ B)

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$T_{AB} = 0$

$T_{BD} = F_B \times \cos\alpha \times \frac{d_B}{2} = 2819.1\ N.m$



- Shoulder fillet @ C:

$r/d = \frac{4}{80} = 0.05$

$\frac{D}{d} = \frac{120}{80} = 1.5$

$\Rightarrow$ from figures A-15-8 and A-15-9

$K_t = 2.1$

$K_{ts} = 1.75$

$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$; $(\sqrt{a})_{bending} = 0.314$

$q_s = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$; $(\sqrt{a})_{torsion} = 0.241$

$\Rightarrow q = 0.86$ and $q_s = 0.89$

$K_f = 1 + q(K_t - 1) \simeq 1.94$

$K_{fs} = 1 + q_s(K_{ts} - 1) \simeq 1.62$

$M_a = M_c = 6400\ N.m$ $\quad \frac{1}{a} = 0$

$M_m = 0$ $\quad \frac{1}{m} = \frac{1}{1} = 2819.1\ N.m$

$A = 2 \times K_f \times M_a = 24\,882\ N.m$

$B = \sqrt{3} \times K_{fs} \times T_m = 7909.9\ N.m$

$S_e = K_a \times K_b \times K_c \times K_d \times K_e \times S_e'$

$S_e' = \frac{1}{2} S_{ut} = \frac{690}{2} = 345\ MPa$

$K_a = a\, S_{ut}^{\ b}$; a $\Rightarrow a = 3.04$ and $b = -0.217 \Rightarrow K_a = 0.736$

$K_b = 1.51\, d^{-0.157} = 0.759$ (1)

$K_c = K_d = K_e = 1$ (1)

$\Rightarrow S_e = 192.7$ MPa (1)

Using DE-Goodman:

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = \frac{\pi(0.08)^3}{16} \times \left(\frac{24912.7}{192.7\times10^6} + \frac{8154}{690\times10^6}\right)^{-1}$$ (2)

$\Rightarrow n = 0.71$

- Keyseat @ B:

endmill $\Rightarrow r/d = 0.02$ (1)

$K_t = 2.14$ (From table 7-2)

$K_{ts} = 3$

$d = D_3 = 80$ mm $\Rightarrow r = 0.02 \times 80 = 1.6$ mm (1)

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}; \quad \sqrt{a} = 0.314$$

$$\Rightarrow q = \frac{1}{1 + \frac{0.314}{\sqrt{1.6}}} = 0.8$$ (1)

$\Rightarrow K_f = 1 + q(K_t - 1) = 1.912$ (1)

$$q_s = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}; \quad \sqrt{a} = 0.241$$

$$\Rightarrow q_s = \frac{1}{1 + \frac{0.241}{\sqrt{1.6}}} = 0.84$$ (1)

$\Rightarrow K_{fs} = 1 + q_s(K_{ts} - 1) = 2.68$ (1)

$M_a = M_B = 5.887\ KN.m = 5887\ N.m$

$M_m = 0$

$T_a = 0$ $\quad T_m = 2819\ N.m$

$\Rightarrow A = 2K_f \times M_a = 22523.36\ N.m$

$B = \sqrt{3} \times K_{fs} \times T_m = 13085.5\ N.m$

Using DE-Goodman:

$$\Rightarrow n = \frac{\pi \times (0.08)^3}{16} \times \left(\frac{22523.36}{192.7 \times 10^6} + \frac{13085.5}{690 \times 10^6} \right)^{-1}$$

$\Rightarrow n = 0.74$

$\Rightarrow$ Shoulder fillet @C is more critical with a safety factor $n = 0.71$

c) Required diameters should be determined based on the most critical location @ the shoulder fillet C.

Using a design factor $n_d = 1$ and using DE-Goodman criterion:

$$d = \left[\frac{16 n_d}{\pi} \left(\frac{A}{S_e} + \frac{B}{S_{ut}} \right) \right]^{1/3} = \left[\frac{16 \times 1}{\pi} \left(\frac{24882}{192.7 \times 10^6} + \frac{7909.9}{690 \times 10^6} \right) \right]^{1/3}$$

$\Rightarrow d = 0.0895\ m = 89.5\ mm$

$\Rightarrow d_{required} = (D_s)_{new} = 90\ mm$

$\frac{d_{new}}{d_{old}} = \frac{90}{80} = 1.125$

$\Rightarrow$ The new dimensions of the shaft for infinite lifetime are:

$D_1 = 56.25\,mm = D_7$

$D_2 = 78.75\,mm = D_6$

$D_3 = 90\,mm = D_5$

$D_4 = 135\,mm$