Formula sheet

(1) Principal Stresses for Plane Stress :

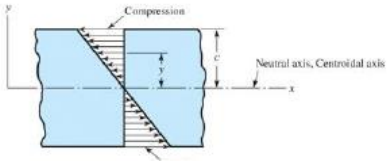
$$\sigma_A, \sigma_B = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(2) Maximum in-plane Plane Stress

$$\tau_{\text{max in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(3) Maximum bending stress

$$\sigma_{\max} = \frac{Mc}{I}$$

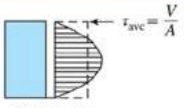
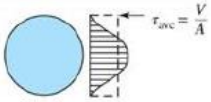
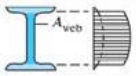


(4) Maximum torsional shear stress

$$\tau_{\max} = \frac{Tr}{J}$$

(5) Maximum Values of Transverse Shear Stress

Table 3–2 Formulas for Maximum Transverse Shear Stress from *VQ/Ib*

Beam Shape	Formula	Beam Shape	Formula
 Rectangular	$\tau_{\max} = \frac{3V}{2A}$	 Hollow, thin-walled round	$\tau_{\max} = \frac{2V}{A}$
 Circular	$\tau_{\max} = \frac{4V}{3A}$	 Structural I beam (thin-walled)	$\tau_{\max} \approx \frac{V}{A_{\text{web}}}$

(6) Maximum Shear Stress Theory

$$n = \frac{S_y/2}{\tau_{\max}}$$

(7) Distortion Energy Theory in plane stress

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

$$\sigma' = \frac{S_y}{n}$$

(8) Coulomb-Mohr Theory

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}$$

(9) Modified-Mohr Theory in Case of plane stress

Quadrant condition			Failure criteria
$\sigma_A \geq \sigma_B \geq 0$			$\sigma_A = \frac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$	and	$\left\|\frac{\sigma_B}{\sigma_A}\right\| \leq 1$	$\sigma_A = \frac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$	and	$\left\|\frac{\sigma_B}{\sigma_A}\right\| > 1$	$\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$
$0 \geq \sigma_A \geq \sigma_B$			$\sigma_B = -\frac{S_{uc}}{n}$

(10) Fracture Mechanics

$$K_I = \beta\sigma\sqrt{\pi a} \qquad n = \frac{K_{Ic}}{K_I}$$

(11)

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200\text{kpsi}\ (1400\text{MPa}) \\ 100\text{kpsi} & S_{ut} > 200\text{kpsi} \\ 700\text{MPa} & S_{ut} > 1400\text{MPa} \end{cases}$$

(12)

$$S_e = k_a k_b k_c k_d k_e S'_e$$

(13)

$$k_a = aS_{ut}^b$$

Surface Finish	Factor a		Exponent b
	S_{ut}, kpsi	S_{ut}, MPa	
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758

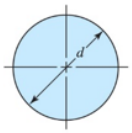
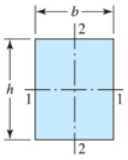
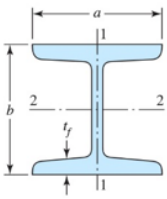
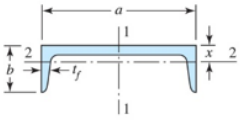
(14)

For rotating shaft. For bending or torsion

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.3 \le d \le 2\,\text{in} \\ 0.91d^{-0.157} & 2 < d \le 10\,\text{in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 7.62 \le d \le 51\,\text{mm} \\ 1.51d^{-0.157} & 51 < 254\,\text{mm} \end{cases}$$

For axial load k_b =1.

For non-rotating member

	$A_{0.95\sigma} = 0.01046d^2$ $d_e = 0.370d$
	$A_{0.95\sigma} = 0.05hb$ $d_e = 0.808\sqrt{hb}$
	$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2} \end{cases}$
	$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b-x) & \text{axis 2-2} \end{cases}$

(15)

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

(16)

$$K_f = 1 + q\,(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_s\,(K_{ts} - 1)$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}}$$

(17)

Bending or axial:

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)S_{ut} + 1.51\left(10^{-5}\right)S_{ut}^2 - 2.67\left(10^{-8}\right)S_{ut}^3 \quad 50 \le S_{ut} \le 250\ \text{kpsi}$$

$$\sqrt{a} = 1.24 - 2.25\left(10^{-3}\right)S_{ut} + 1.60\left(10^{-6}\right)S_{ut}^2 - 4.11\left(10^{-10}\right)S_{ut}^3 \quad 340 \le S_{ut} \le 1700\ \text{MPa}$$

` Torsion:

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)S_{ut} + 1.35\left(10^{-5}\right)S_{ut}^2 - 2.67\left(10^{-8}\right)S_{ut}^3 \quad 50 \le S_{ut} \le 220 \text{ kpsi}$$
$$\sqrt{a} = 0.958 - 1.83\left(10^{-3}\right)S_{ut} + 1.43\left(10^{-6}\right)S_{ut}^2 - 4.11\left(10^{-10}\right)S_{ut}^3 \quad 340 \le S_{ut} \le 1500 \text{ MPa}$$

(18)

$$S_f = aN^b$$

$$a = \frac{\left(fS_{ut}\right)^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$$

(19)

$$f = 1.06 - 2.8\left(10^{-3}\right)S_{ut} + 6.9\left(10^{-6}\right)S_{ut}^2 \quad 70 < S_{ut} < 200 \text{kpsi}$$
$$f = 1.06 - 4.1\left(10^{-4}\right)S_{ut} + 1.5\left(10^{-7}\right)S_{ut}^2 \quad 500 < S_{ut} < 1400 \text{ MPa}$$

(20)

$$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b}$$

(21)

Goodman criterion

Failure criterion: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$

Design equation: $n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} \quad \sigma_m \ge 0$

(22)

Gerber criterion

Failure criterion: $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1$

Design equation: $n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left(\frac{\sigma_a}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \quad \sigma_m \ge 0$

(23)

Gerber criterion

$$\text{Failure criterion:} \qquad \frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_y}=1$$

$$\text{Design equation:} \quad n=\left(\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_y}\right)^{-1} \qquad \sigma_m \geq 0$$

(24)

ASME Elliptic

$$\text{Failure criterion:} \qquad \left(\frac{\sigma_a}{S_e}\right)^2+\left(\frac{\sigma_m}{S_y}\right)^2=1$$

$$\text{Design equation:} \quad n_f=\left[\left(\frac{\sigma_a}{S_e}\right)^2+\left(\frac{\sigma_m}{S_y}\right)^2\right]^{-1/2} \qquad \sigma_m \geq 0$$

(25)

For case of a shaft with bending stresses, torsional shear stresses, and axial stress, the von Mises stress is of the form

$$\sigma'=\left(\sigma_x^2+3\tau_{xy}^2\right)^{1/2}$$

$$\sigma_a'=\left\{\left[\left(K_f\right)_{\text{bending}}\left(\sigma_{a0}\right)_{\text{bending}}+\left(K_f\right)_{\text{axial}}\left(\sigma_{a0}\right)_{\text{axial}}\right]^2+3\left[\left(K_{fs}\right)_{\text{torsion}}\left(\tau_{a0}\right)_{\text{torsion}}\right]^2\right\}^{1/2}$$

$$\sigma_m'=\left\{\left[\left(K_f\right)_{\text{bending}}\left(\sigma_{m0}\right)_{\text{bending}}+\left(K_f\right)_{\text{axial}}\left(\sigma_{m0}\right)_{\text{axial}}\right]^2+3\left[\left(K_{fs}\right)_{\text{torsion}}\left(\tau_{m0}\right)_{\text{torsion}}\right]^2\right\}^{1/2}$$

(26)

$$n_y=\frac{S_y}{\sigma'_{\max}} \qquad\qquad \sigma'_{\max}\approx\sigma_a'+\sigma_m'$$

(27)

Constant-Life in case of fluctuating stress $\sigma_m > 0$ $\qquad N=\left(\frac{\sigma_{ar}}{a}\right)^{1/b}$

<u>Using Goodman criterion</u>

$$\sigma_{ar}=\frac{\sigma_a}{1-\sigma_m/S_{ut}}$$

(28)

Shaft Stresses (Shafts subjected to transverse loading and torque)

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_f \frac{M_m c}{I}$$

$$\tau_a = K_{fs} \frac{T_a r}{J} \qquad \tau_a = K_{fs} \frac{T_m r}{J}$$

For round shafts

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32 M_m}{\pi d^3}$$

$$\tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16 T_m}{\pi d^3}$$

Von Mises stresses

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

For simpler form

$$A = \sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2}$$

$$B = \sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}$$

DE-Goodman

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1}$$

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3}$$

DE-Gerber

$$\frac{1}{n}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{-1/2}\right\}$$

$$d=\left(\frac{8nA}{\pi S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{-1/2}\right\}\right)^{1/3}$$

(29)

Shaft Stresses (Shafts subjected to transverse loading and torque)

$$\sigma'_{\max}=\left[\left(\sigma_m+\sigma_a\right)^2+3\left(\tau_m+\tau_a\right)^2\right]^{1/2}$$

$$=\left[\left(\frac{32K_f\left(M_m+M_a\right)}{\pi d^3}\right)^2+3\left(\frac{16K_{fs}\left(T_m+T_a\right)}{\pi d^3}\right)^2\right]^{1/2}$$

(30)

Estimating Stress Concentrations

Table1: First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts}

	Bending	Torsional	Axial
Shoulder fillet—sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet—well rounded (r/d = 0.1)	1.7	1.5	1.9
End-mill keyseat (r/d = 0.02)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

(31)

$$d_{\text{new}}=d_{\text{old}}\left|\frac{n_d y_{\text{old}}}{y_{\text{all}}}\right|^{1/4}$$

$$d_{\text{new}}=d_{\text{old}}\left|\frac{n_d\left(dy/dx\right)_{\text{old}}}{\left(\text{slope}\right)_{\text{all}}}\right|^{1/4}$$

Table 2: Typical Maximum Ranges for Slopes and Transverse Deflections

Slopes	
Tapered roller	0.0005 to 0.0012 rad
Cylindrical roller	0.0008 to 0.0012 rad
Deep-groove ball	0.001 to 0.003 rad
Spherical ball	0.026 to 0.052 rad
Self-align ball	0.026 to 0.052 rad
Uncrowned spur gear	<0.0005 rad
Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in

Appendix

Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

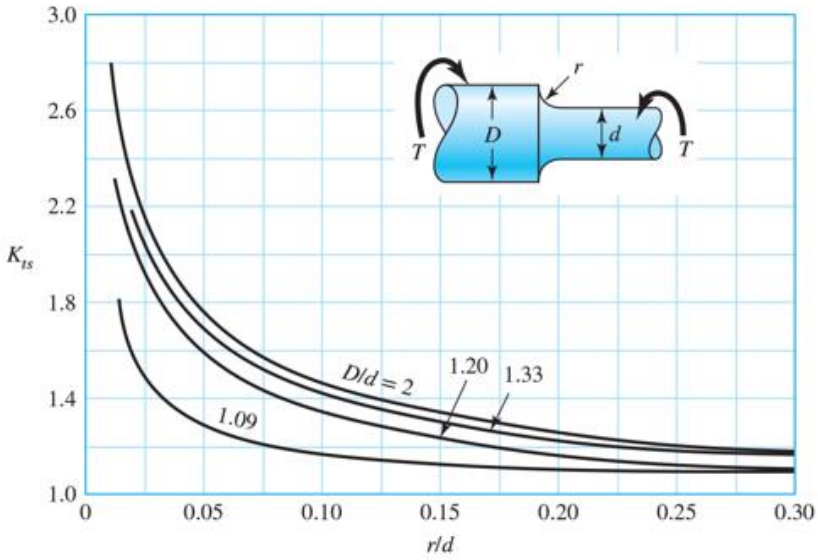


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

